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THE MEASUREMENT OF NUCLEAR LIFETIMES
BY A DIRECT ELECTRONIC METHOD

by



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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled THE MEASUREMENT OF NUCLEAR LIFETIMES BY A DIRECT ELECTRONIC METHOD submitted by Joshua Daniel Panar in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

Efforts to improve timing resolution from various scintillator-photomultiplier combinations are described and the results of this work are applied to the direct electronic measurement of nuclear lifetimes for levels of ^{45}Sc and ^{45}Ti . A lifetime of 565 ± 20 psec has been found in ^{45}Sc from a level between 2.8 and 3.2 MeV excitation energy excited by the $^{45}\text{Sc}(p,p')^{45}\text{Sc}^*$ reaction. In addition, upper limits of 180 psec have been set on the mean lifetimes of levels of ^{45}Ti up to 1.52 MeV excitation energy strongly excited by the $^{45}\text{Sc}(p,n)^{45}\text{Ti}^*$ reaction.

A review of various lifetime measurement techniques and improvements to the technique are also included.

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I. INTRODUCTION

This thesis will describe work done on improving the technique of measuring nuclear lifetimes in the nanosecond and subnanosecond range by a direct electronic method. The method, a form of delayed coincidence, is described in detail in section III, but will be briefly explained here to provide a degree of familiarity with the material.

A nuclear state is excited by regularly spaced beam pulses from a Van de Graaff accelerator. The radioactive decay spectrum is obtained by measuring the time interval between the beam pulses and the output pulses from a scintillation detector. Then, from the radioactive decay law

$$\frac{dN}{dt} = -\lambda N \quad [1]$$

where N is the number of still excited nuclei surviving after the time t and λ is the decay constant, the mean life τ_m of the level is found

$$\tau_m = \frac{1}{\lambda} \quad \text{and} \quad N = N_i e^{-\lambda t} \quad [2.a]$$

where N_i is the number of excited nuclei at time $t = 0$.

When the number of gamma rays that enter a detector in a given time period Δt is set equal to N_0 and the number that enters the same detector under the same conditions (same Δt , same experimental setup) at a time t later is set equal to N , in the limit that $\Delta t \rightarrow 0$, the mean life of the gamma emitting level is given by

$$\tau_m = \frac{t}{\ln N_0 - \ln N} .$$

τ_m is of interest to nuclear physics as its theoretical value is extremely sensitive to the nuclear model used to calculate it. Thus

comparisons of theoretical and experimental values will provide sensitive tests of nuclear models. This will be shown in section II.

This thesis will be divided into seven sections. The next section presents a review of the theory of nuclear decay as applied to gamma radiation and the third section gives a review and a comparison of the various methods of measuring nuclear lifetimes. The fourth section describes the experiments carried out as part of this project, both on the bench and using the accelerator, while the fifth section contains conclusions and discussion. Appendices are given in section six. The final section will consist of a glossary. A list of the references follows the final section.

II. THEORY

An excited nucleus can de-excite by particle emission, β decay, gamma emission, internal conversion or fission. It is the third method which concerns us here and this section will present a review of the theory of electromagnetic radiation with particular emphasis on the transition probability which is inversely proportional to the nuclear lifetimes. This is done to provide the general background material relevant to this topic as well as to specifically relate the lifetime information to nuclear structure theory.

Electromagnetic radiation is classed by multipole order L where L is the angular momentum in units of $\hbar$ carried off by each emitted quantum. Each multipole order has two classes of radiation, the electric 2^L pole (EL) and the magnetic 2^L pole (ML). While these differ with respect to parity and classically they refer to radiation emitted by electric or magnetic 2^L poles, the physical difference between electric and magnetic multipole fields can be deduced from the following equation

$$\underline{r} \cdot \underline{E}_{LM}^M = \underline{r} \cdot \underline{H}_{LM}^E = 0 \quad [3]$$

which is derived in Appendix I.¹)* This equation states that the electric multipole field has no non-vanishing radial components of the magnetic field $\underline{H}$ (the electric multipole field does have non-vanishing

*In equation 3 the superscript E and M refer to the electric or magnetic nature of the radiation while the subscript M is the Z component of the angular momentum, L .

radial components of the electric field $\underline{E}$), while the magnetic multipole field has non-vanishing radial components of $\underline{H}$ but not of $\underline{E}$.

The conservation of angular momentum and parity of the nuclear system plus γ -rays requires

$$|J_i - J_f| \leq L \leq J_i + J_f \quad [4]$$

and
$$\Delta\pi = \pi_i / \pi_f = (-1)^L \quad \text{for EL radiation} \quad [5.a]$$

$$= (-1)^{L-1} \quad \text{for ML radiation} \quad [5.b]$$

where J_i and J_f are the initial and final angular momenta of the nuclear states (parities π_i, π_f), and L is the angular momentum carried off by the emitted quantum. $\Delta\pi = +1$ denotes no parity change while $\Delta\pi = -1$ denotes a change of parity. In addition, no radiation of multipole order 0 can be emitted. This is a consequence of the transverse nature of electromagnetic radiation and can be seen from equation I.21 in Appendix I.

In a study of the gamma radiation transition probability the single particle model is first considered. In particular, the transition probability for a single non-relativistic proton moving in a potential due to all the other particles in the nucleus is evaluated. This is not a correct picture of the nucleus, but as transition probabilities of more realistic models can be expressed in terms of the results of this simpler view, we proceed.

The quantum mechanical transition probability $T_{i \rightarrow f}$ for any process is given by

$$T_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle f | H' | i \rangle|^2 \frac{dN}{dE} \quad [6]$$

where H' is the perturbing interaction and $\langle f | H' | i \rangle$ is the first order matrix element of the interaction between the wave-function of the initial state i and the final state f . $\frac{dN}{dE}$ denotes the density of final states. This equation is developed in Appendix II. In a non-relativistic treatment the total Hamiltonian of the proton and the electromagnetic field is

$$H = \frac{(\underline{P} - (e/c)\underline{A})^2}{2m} + V(\underline{r}) - \mu_p \frac{e\hbar}{2mc} \underline{\sigma} \cdot \underline{H} + \sum_{\lambda} n_{\lambda} \hbar \omega_{\lambda} \quad [7]$$

where m is the proton mass, $\mu_p (e\hbar/2mc)$ its magnetic moment, $\underline{H}$ is the magnetic field, $\underline{\sigma}$ the Pauli spin vector, and $\underline{P}$ is the momentum operator $-i\hbar \text{grad}$. In the lowest order of interaction (single quantum emission), this becomes

$$H'(A) = -\frac{e}{mc} \frac{\underline{P} \cdot \underline{A} + \underline{A} \cdot \underline{P}}{2} - \mu_p \frac{e\hbar}{2mc} \underline{\sigma} \cdot \underline{H} \quad [8]$$

The Hamiltonian is taken between an initial state consisting of a proton in state i and no quanta and a final state f with one quantum characterized by quantum numbers λ . From equations I.4 and I.11 of Appendix I

$$\langle f, 1_{\lambda} | H'(A) | i, 0 \rangle = \left(\frac{\hbar}{2\omega} \right)^{1/2} \langle f | H'(\underline{A}_{\lambda}^*) | i \rangle \quad [9]$$

where

$$\omega = \frac{E_i - E_f}{\hbar} \quad [10]$$

Note that the matrix element on the right hand side of equation 9 is independent of $q_{\lambda}, q_{\lambda}^*$ (the creation and annihilation operators of Appendix I), and time.

The transition probability term for the density of states in a spherical volume of large radius R_0 is

$$\frac{dN}{dE} = \frac{R_0}{\pi \hbar c} \quad [11]$$

Thus

$$T_{i \rightarrow f}^\lambda = \frac{R_0}{\hbar c \omega} |\langle f | H'(\underline{A}_\lambda^*) | i \rangle|^2. \quad [12]$$

Applying equations I.22 to equation 8

$$H'(\underline{A}_{LM}^{E*}) = -\frac{C_L e}{m \omega} \underline{P} \cdot (\text{curl } \underline{L} u_{LM})^* - \frac{C_L \mu e \hbar}{2mc^2} \omega \underline{\sigma} \cdot (\underline{L} u_{LM})^* \quad [13.a]$$

$$H'(\underline{A}_{LM}^{M*}) = \frac{C_L i e \underline{P} \cdot (\underline{L} u_{LM})^*}{mc} \cdot \frac{C_L i \mu e \hbar}{2mc} \underline{\sigma} \cdot (\text{curl } \underline{L} u_{LM})^* \quad [13.b]$$

At this point the approximation that the nuclear radius a is small compared to the wavelength of the radiation,

$$\frac{\omega a}{c} \ll 1 \quad [14]$$

is introduced, allowing

$$u_{LM} = \frac{(\omega r/c)^L}{(2L+1)!!} Y_{LM}(\theta, \phi) \quad [15]$$

(from equation II.15 and

$$j_L(k_\lambda r) \xrightarrow{k_\lambda r \rightarrow 0} \frac{(k_\lambda r)^L}{(2L+1)!!} \quad [16]$$

Stech²⁾ has proven the following two relations which are used to further simplify the expressions for the transition probabilities:

$$(\text{curl } \underline{L} u_{LM})^* = i[\underline{r} \Delta u_{LM}^* - (\underline{r} \times \text{grad} + 2)\text{grad } u_{LM}]^* \quad [17.a]$$

$$\approx -i(L+1)[\text{grad } u_{LM}]^* \quad [17.b]$$

and

$$\begin{aligned} \langle f | \underline{p} \cdot (\text{grad } u_{LM})^* | i \rangle &= - \frac{i m}{\hbar} \langle f | \frac{p^2}{2m} u_{LM} - u_{LM} \frac{p^2}{2m} | i \rangle \\ &= - i m \omega \langle f | u_{LM}^* | i \rangle \end{aligned} \quad [18]$$

Using these and introducing the quantities M_{LM}^σ ($\sigma \equiv E$ or M)

$$\langle f | H' (A_{LM}^{\sigma*}) | i \rangle = C_L \frac{L+1}{(2L+1)!!} \left(\frac{\omega}{c}\right)^L \langle f | M_{LM}^\sigma | i \rangle \quad [19]$$

where

$$M_{LM}^E = e r^L Y_{LM}^* - \frac{i \mu_p e \hbar \omega}{2mc} (L+1)^{-1} \underline{\sigma} \times \underline{r} \cdot [\text{grad}(r^L Y_{LM})]^* \quad [20.a]$$

$$M_{LM}^M = \frac{e \hbar}{mc} \frac{1}{L+1} \underline{L} \cdot [\text{grad}(r^L Y_{LM})]^* + \frac{e \hbar}{2mc} \mu_p \underline{\sigma} \cdot [\text{grad}(r^L Y_{LM})]^* \quad [20.b]$$

Using the expression for C_L from equation I.25

$$T_{i \rightarrow f}^{(\sigma LM)} = \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{1}{\hbar} \left(\frac{\omega}{c}\right)^{2L+1} |\langle f | M_{LM}^\sigma | i \rangle|^2 \quad [21]$$

Before proceeding to derive the single particle transition rates (in terms of which the more realistic rates are expressed) several observations may be made. While the selection rules given by equations 4 and 5 still hold, equations 20 imply an additional criterion on the orbital angular momenta (l_i, l_f) if the particle is moving in a central velocity-independent potential,

$$|l_i - l_f| \leq L \leq l_i + l_f \quad [22]$$

Equations 20 and 21 can also be used to show that if $(\omega a/c) \ll 1$, then transitions for a single proton will tend to proceed by the lowest multiple order as follows. By taking the orders of magnitude for the M_{LM}^σ ,

$$M_{LM}^E = o(e a^L) \quad [23.a]$$

$$M_{LM}^M = o\left[\left(\frac{\hbar}{mca}\right)(ea^L)\right], \quad [23.b]$$

orders of magnitude for the transition probabilities can be expressed as follows

$$T_{i \rightarrow f}^{(EL)} = o\left[\omega\left(\frac{e}{\hbar c}\right)^2 (\omega a/c)^{2L}\right] \quad [24.a]$$

$$T_{i \rightarrow f}^{(ML)} = o\left[\omega\left(\frac{e}{\hbar c}\right)^2 (\hbar/mca)^2 (\omega a/c)^{2L}\right] \quad [24.b]$$

The ratios of the two lowest terms are:

$$1) \quad |j_i - j_f| = L(\geq 1), \quad \Delta\pi = (-1)^L$$

$$\frac{T^{(M(L+1))}}{T^{(EL)}} = o\left[(\hbar\omega/mc^2)^2\right] \quad [25.a]$$

$$2) \quad |j_i - j_f| = L(\geq 1), \quad \Delta\pi = (-1)^{L-1}$$

$$\frac{T^{(E(L+1))}}{T^{(ML)}} = o\left[(ma^2/\hbar)^2\right] \quad [25.b]$$

In general the ratios of equations 25 will be much less than unity and the radiative transitions will in fact proceed by the lowest multipole order. In actual practice, many transitions involve the two lowest orders and more complex models do, in fact, account for this. It is in deducing the single particle transition probabilities that the sensitivity of the study of nuclear lifetimes to nuclear structure becomes evident. The dependence of the transition probability on the form of the initial and final state wavefunctions makes the study of nuclear lifetimes an excellent check on the validity of these model-dependent wavefunctions. This is seen as follows. If the proton is assumed to be

moving in a central potential each wavefunction ψ_i and ψ_f can be written as a product of a radial function and an angular function. The radial functions R_i and R_f depend upon the details of the potential, while the two component functions $\theta_{l_i, j_i}^{m_i}$ and $\theta_{l_f, j_f}^{m_f}$ contain angular and spin variables only. This allows the electric multipole matrix element to be rewritten

$$\langle f | M_{LM}^E | i \rangle = e \int_0^\infty R_f r^L R_i r^2 dr \int_{4\pi} \theta_{l_f, j_f}^{m_f*} Y_{LM}^* \theta_{l_i, j_i}^{m_i} d\Omega \quad [26.a]$$

where the contribution from the spin term in equation 20.a has been neglected as insignificant. However, the magnetic multipole matrix element is much more complex, as the spin term is actually more important than the orbital term and so a simplifying special case is considered, $|j_i - j_f| = L$ and $|l_i - l_f| = L - 1$. With these criteria, equation 20.b gives

$$\langle f | M_{LM}^M | i \rangle = \frac{e\hbar}{2mc} (\mu_p - \frac{1}{L+1}) \langle f | \underline{\sigma} \cdot \text{grad}(r^L Y_{LM}^*) | i \rangle \quad [26.b]$$

The transition probability is summed over the m substates of the final state and is averaged over the m substates of the initial state to give a total transition probability

$$T_{i \rightarrow f}^{(\sigma L)} = (2j_i + 1)^{-1} \sum_{m_i} \sum_{m_f} \sum_M T_{i \rightarrow f}^{(\sigma LM)} \quad [27]$$

This is evaluated with the help of a statistical factor

$$S(j_i, L, j_f) = 4\pi (2j_i + 1)^{-1} \sum_{m_i} \sum_{m_f} \sum_M \left| \int_{4\pi} \theta_{l_f, j_f}^{m_f*} Y_{LM}^* \theta_{l_i, j_i}^{m_i} d\Omega \right|^2 \quad [28.a]$$

$$= (2j_f + 1) [C(j_i, j_f, L, 1/2, -1/2, 0)]^2 \quad [28.b]$$

where C is a Clebsh-Gordan Coefficient. The values of $S(j_i, L, j_f)$ are listed in tables.¹⁾ Using this statistical factor, the transition probabilities become

$$T_{i \rightarrow f}^{(EL)} = \frac{2(L+1)}{L[(2L+1)!!]^2} \omega \left(\frac{e}{\hbar c} \right)^2 (\omega a/c)^{2L} \left(\int_0^\infty R_f(r/a)^L R_i r^2 dr \right)^2 S(j_i, L, j_f) \quad [29.a]$$

$$T_{i \rightarrow f}^{(ML)} = \frac{2(L+1)}{L[(2L+1)!!]^2} \omega \left(\frac{e}{\hbar c} \right)^2 (\omega a/c)^{2L} (\hbar/mca)^2 \left(\mu_p^L - \frac{L}{L+1} \right)^2 \left(\int_0^\infty R_f(r/a)^{L-1} R_i r^2 dr \right)^2 S(j_i, L, j_f) \quad [29.b]$$

By noting that $T_{i \rightarrow f}^{(\sigma L)} = \frac{1}{\tau_m}$, sensitive tests of the form of the R 's may be made providing clues to nuclear structure. If the radial functions are assumed to be constant inside the nuclear volume ($r < a$) and vanish outside ($r > a$),

$$\int_0^\infty R_f(r/a)^L R_i r^2 dr = 3(L+3)^{-1}. \quad [30]$$

With this assumption, the single particle transition probabilities for a single proton become

$$T_{sp}^{(EL)} = \frac{4.4(L+1)}{L[(2L+1)!!]^2} \left(\frac{3}{L+3} \right)^2 \left(\frac{\hbar\omega}{197} \right)^{2L+1} a^{2L} S(j_i, L, j_f) \times 10^{21} \text{sec}^{-1} \quad [31.a]$$

and

$$T_{sp}^{(ML)} = \frac{0.19(L+1)}{L[(2L+1)!!]^2} \left(\frac{3}{L+2} \right)^2 \left(\mu_p^L - \frac{L}{L+1} \right)^2 \left(\frac{\hbar\omega}{197} \right)^{2L+1} a^{2L-2} S(j_i, L, j_f) \times 10^{21} \text{sec}^{-1} \quad [31.b]$$

where $\hbar\omega$ is a MeV, and a is in fermis. Table I gives the estimates for various multipolarities.

TABLE I

Multipolarity	T_{sp} in sec^{-1}		
E1	1.0×10^{14}	$A^{2/3}$	$E_{\gamma}^3 S$
M1	2.9×10^{13}		$E_{\gamma}^3 S$
E2	7.4×10^7	$A^{4/3}$	$E_{\gamma}^5 S$
M2	8.4×10^7	$A^{2/3}$	$E_{\gamma}^5 S$
E3	3.4×10^1	A^2	$E_{\gamma}^7 S$
M3	8.7×10^1	$A^{4/3}$	$E_{\gamma}^7 S$
E4	1.1×10^{-5}	$A^{8/3}$	$E_{\gamma}^9 S$
M4	4.8×10^{-5}	A^2	$E_{\gamma}^9 S$

For single neutron transitions the electric transition probabilities are smaller than the proton transition probabilities by a factor

$$Z/A^L \quad [32.a]$$

and the magnetic transition probabilities are slightly smaller than the corresponding probabilities for electric transitions, the ratio being

$$\left[\frac{\mu_N}{\mu_p - (L+1)^{-1}} \right]^2 \quad [32.b]$$

Mention should be made of many particle configurations for which a reduced transition probability is defined

$$B(\sigma L, J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \sum_{m_i} \sum_{m_f} \sum_M \left| \langle f | \sum_p M_{LM}(p) | i \rangle \right|^2. \quad [33]$$

This gives for the transition probability

$$T_{i \rightarrow f}^{(\sigma L)} = \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{1}{\hbar} (\omega/c)^{2L+1} B(\sigma L, J_i \rightarrow J_f) . \quad [34]$$

III. REVIEW OF METHODS OF LIFETIME MEASUREMENT

The purpose of this section is to review the methods used in the study of nuclear lifetimes. Such a review has been carried out by Schwarzschild and Warburton³⁾ and the techniques dealt with in their paper are treated below along with a brief mention of Resonance Absorption and Fluorescence. A division is made between methods employing a form of Doppler shift, and those which do not. The former class includes the Doppler Shift Attenuation Method (DSAM), Resonance Methods and the Recoil-Distance method. Delayed coincidence methods including the Direct Electronic Timing method make up the latter category.

The first Doppler shift technique is the Doppler Shift Attenuation Method (DSAM). This method has been discussed in detail by ^{A.E.} ~~E.A.~~ Litherland et al.⁴⁾ and E.K. Warburton et al.⁵⁾. It involves exciting a target nucleus which particle decays to an excited state that subsequently gamma decays. After the particle decay, the excited residual nucleus recoils in accordance with momentum conservation and is slowed down at a known rate in the target backing material. The gamma rays emitted are Doppler shifted to a degree dependent upon the velocity of the recoiling nucleus at the time of gamma emission. By measuring this shift and by knowing the recoil velocity as a function of time, the time of emission and hence the lifetime can be determined.

The method is effective for lifetimes in the range 10^{-14} to 10^{-11} sec⁴⁾ for solid targets and the range is extended to 10^{-10} sec⁶⁾

for gas targets. While this implies that the choice of target is severely limited for lifetimes longer than 10^{-11} sec, the method has the advantage of being relatively quick and easy to carry out. However, the precise knowledge required of the recoil ion velocity as a function of time limits the accuracy of the DSAM to the order of 15 - 20%⁶). Other effects such as Doppler broadening and inhomogeneities in the target material and backings also give rise to uncertainties. But above all, the method while effective, is an indirect one and it is with this point in mind that attempts are being made to extend the more direct methods into this lifetime region.

The second Doppler shift method to be studied is really two variations of the same theme, Resonance Absorption and Resonance Fluorescence. In atomic physics resonance absorption is a well known phenomenon. The absorption-line spectrum where certain frequencies of an impinging light beam excite the orbital electrons of a gaseous material and are thereby lost to the beam is an old analytical tool often used to identify a material in question. A related phenomenon is fluorescence or resonance scattering in which the impinging light causes resonant stimulated emission of radiation. Although Kuhn⁸) predicted as early as 1929 that these resonant features would occur in nuclei, they have been observed only within the past two decades initially by Moon⁹). This is because the recoil of the emitting nucleus takes energy and Doppler shifts the frequency of the emitted radiation into a frequency which cannot be reabsorbed by a second similar medium.

$$f_o = f_s \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \quad [35]$$

where f_o is the frequency of the radiation as seen by the second nucleus which would absorb the radiation if the shift were not taking place, and f_s ^{is the} ~~with~~ natural emission frequency.

These phenomena can be used to measure nuclear lifetimes in the following way. The spectral lines have a natural width in their frequencies $\delta\omega$ which is proportional to the mean life τ of a nuclear state according to the relation

$$\Gamma\tau = \hbar \quad [36.a]$$

and

$$\Gamma = \hbar\delta\omega \quad [36.b]$$

where Γ is the level width in energy units, and $\hbar$ is Planck's constant divided by 2π . Thus if the level width can be found, the lifetimes of the state in question will be known. Table II lists approximate values of Γ with corresponding values of τ .

TABLE II

<u>$\Gamma(\text{ev})$</u>	<u>$\tau(\text{sec})$</u>	<u>$\Gamma(\text{eV})$</u>	<u>$\tau(\text{sec})$</u>
6×10^{-10}	10^{-6}	6×10^{-1}	10^{-15}
6×10^{-7}	10^{-9}	6×10^2	10^{-18}
6×10^{-4}	10^{-12}	6×10^5	10^{-21}

$\delta\omega$ can be found if two pieces of the same material are set up so that one piece which is excited and emits gamma rays can be made to move towards a non-excited piece which acts as a resonant scatterer or absorber. This motion of the source medium can be made fast enough mechan-

ically so that it creates a Doppler shift of the emitted gamma ray which will be exactly equal and opposite to the recoil Doppler shift, thereby allowing the 'target' piece to act as a resonant scatterer or absorber. By measuring the intensity of the scattering or absorbing as a function of the source medium velocity (hence as a function of energy due to Doppler shifting), the degree of the overlap of the shifted gamma ray's natural energy spread and the energy level in question (the scattering or absorption cross section) gives the width of the level in terms of energy units. In practice the method is limited in several ways. At the outset, the Doppler shift energies must be resolveable implying that this method is applicable only to nuclear lifetimes in the 10^{-15} sec region and shorter (see table II). Also, the isotope in question must not only have a substantial natural abundance to be useful as a scatterer, but it must also be available in an excited state following the decay of its radioactive parent. One of the more formidable drawbacks is that the velocity required for the correct Doppler shift be readily attainable by mechanical means (up to 10^5 cm/sec). These considerable problems are in most cases insurmountable leaving the technique, at best, a demonstration of the correctness of the theory of resonance fluorescence.

In 1958, Mössbauer¹⁰⁾ discovered another method of applying resonance effects to lifetimes studies, in this case recoilless resonance absorption rather than the resonance fluorescence specifically studied by Moon. Mössbauer reasoned that if the emitting nucleus had a very large mass it would receive the recoil momentum but almost no energy from a gamma transition, thus almost completely eliminating the gamma ray Doppler shift. In fact, an infinitely heavy nucleus does not exist,

but a nucleus bound in a rigid crystal will act as an infinite mass and recoilless transitions can thus take place. It should be noted that the term 'recoilless' refers to energy only and not to the momentum transferred to the emitting nucleus. Under these conditions, small relative velocities between the source and absorber bodies may be used to measure the absorption cross section giving the level width and hence the lifetime involved. While this eliminated the high velocity problem of Moon's technique, the other drawbacks mentioned above still apply.

In addition, a detailed account of the theory of this method¹¹⁻¹⁴⁾ introduces an important factor, the Debye-Waller factor, into the final result and this can be measured to within not better than 20%¹⁵⁾¹⁶⁾. The cryogenic temperatures required for the experiment and accurate low velocity drives are two more features which make this method difficult to use for lifetime studies. As a result, extensive work has not been carried out with this method.

The final Doppler shift technique to be reviewed is the Recoil-Distance method. It is intended to be used in the region between the DSAM region and the direct electronic timing region, i.e. from 10^{-9} to 5×10^{-12} seconds. The technique is described in detail by Alexander and Allen¹⁷⁾ but is briefly presented as follows. A heavy ion beam is used to strike a thin target back-mounted on a thin supporting foil causing the residual nuclei to recoil freely into a vacuum with velocity v . Since the characteristic slowing down time for heavy ions in a metal is typically 3×10^{-13} sec, a metal plate on a moveable plunger placed at a distance $s = vt$ from the target will stop the recoiling ions in a time which is very short compared to their nuclear lifetimes. The in-

tensity of the gamma rays from nuclei slowed down in the plate will thus give the number of nuclei surviving for a time t . The gamma rays from nuclei which are stopped (i.e. ions that have reached the metal plate with lifetimes $> t$) are separated from those which decay in flight by using their Doppler shift separation in energy. By varying the distance s , a decay spectrum is built up as a function of t . While this method can use a variety of solid (and gas¹⁸) targets in a fairly wide time range, not only does it suffer from being an indirect method but only heavy ions can be used as projectiles and Doppler broadening of the shifted line reduces accuracies to about 10%¹⁷).

The other basic method of measuring nuclear lifetimes is by the delayed coincidence technique and its variations, of which our method is one. In its simplest form a radioactive parent nucleus with a daughter possessing a mean gamma decay lifetime in the range 10^{-7} to 10^{-10} sec is considered. Two detectors are used, one starting a clock at the time of the creation of the daughter nucleus by recording an emitted particle from decay of the parent nucleus and the second stopping the clock by recording the gamma ray from the decay of the daughter nucleus. By taking many of these measurements an exponential decay spectrum is built up of gamma rays v.s. time, and the mean life τ is found. While this technique is standard and has been used a great number of times (cf.¹⁹) it is limited in the choice of nuclei to be studied by the parent-daughter requirement. It is in overcoming this that the direct electronic method was developed²⁰). A pulsed beam is made to strike a target composed of either the nuclei to be studied ($M_1(m_2, m_2')M_1^*$ reactions) or a nucleus which can be transmuted into the required nucleus ($M_1(m_2, m_3)M_4^*$ reactions). The time of excitation of the final nucleus is

given by having the arrival time of a beam pulse on the target start a clock, while the emitted gamma ray is used to stop the clock. After many beam pulses an exponential decay spectrum is collected and this is analysed to give τ , the lifetime. Analysis is generally carried out by two methods, least square fitting to either the exponential decay curve, and centroid shift (see below). These methods are quite simple when only one gamma ray with a lifetime greater than the system resolution (cf. section IV) is present. Both methods require a 'prompt' peak to be taken, i.e. a spectrum of gamma rays from levels whose lifetimes are so short that their exponential decay tails are completely masked by the resolution of the detector system. The centroid shift method due to Bay²¹⁾ involves the moments of the distribution of the gamma rays with respect to time where the n^{th} moment is given by

$$M^{(n)}[f(x)] = \int_{-\infty}^{\infty} x^n f(x) dx . \quad [37]$$

Bay considered only the first moment to calculate the lifetime

$$\tau = \frac{M^{(1)}[N_g(t)]}{I} - \frac{M^{(1)}[v_g(t)]}{I} \quad [38]$$

where $N_g(t)$ is the delay curve distribution and $v_g(t)$ is the prompt curve distribution where both are normalized such that

$$\int_{-\infty}^{\infty} N_g(t) dt = \int_{-\infty}^{\infty} v_g(t) dt = I , \quad [39]$$

I thus being the area under the curves. Holland and Lynch²⁰⁾ tried using higher moments as well but found that less intense lifetime contributions from other gamma rays tended to disrupt the calculations.

The least squares method involves fitting either a calculated ex-

ponential curve to the decay spectrum to find values of N_0 and λ (equation 2.a) or fitting a straight line to the natural logarithm of the curve (equation 2.b). In either case it is usual for the prompt peak to be subtracted from the data so that any slope present in the logarithm plot is due to the 'lifetime' gamma ray only.

The lifetime region of this method can be extended in the 'long time length' direction almost indefinitely and is limited in the short lifetime direction by the fact that the decay tail must have a slope which is less than that of the system resolution peak if the decay tail is to be seen. Thus a great deal of effort has gone into making the system resolution or prompt peak slope as narrow and as steep-sided as is possible (cf. section IV). The limit of our work along this line is now in the 10^{-10} sec region.

Obvious advantages of this method are the variety of target materials and constructions allowed and the fact that this method, more than any other, is a direct measurement of the nuclear lifetimes.

IV. EXPERIMENTAL

A simplified diagram of the experimental setup is given in figure 1.a and is explained as follows. The University of Alberta Van de Graaff accelerator is used to provide a pulsed beam of protons at 1 MHz and 0.4 nsec by a Mobley bunching system. This system has been described in detail in previous publications²²). A pulse of the beam first passes through the capacitive pickoff to produce a signal which is delayed and used to create the stop signal for the timing clock. The beam pulse then strikes and excites the target which emits a gamma ray after a time characteristic of the target material.

In the experiment, gamma radiation is detected with scintillators composed of a phosphorescent material in a supporting medium. In the scintillator the incident gamma ray Compton scatters on an electron and the latter slows down by exciting the atoms of the medium and the phosphorescent material. Thus the maximum amount of light emitted for an incident gamma ray is proportional to the maximum Compton recoil energy. This emitted light is transformed into an electronic pulse by a photomultiplier tube which consists of a sensitive photocathode (which gives off electrons when struck by the light) followed by electron multiplication stages. Thus when a gamma ray strikes the scintillator a pulse is created which is used to feed the fast start discriminator which in turn produces a signal used to start the clock. The size of the clock output signal is ^{inversely} proportional to the length of the delay between excitation and decay of the target nuclei (the nuclear lifetime) and this analogue signal is converted into digital form which the

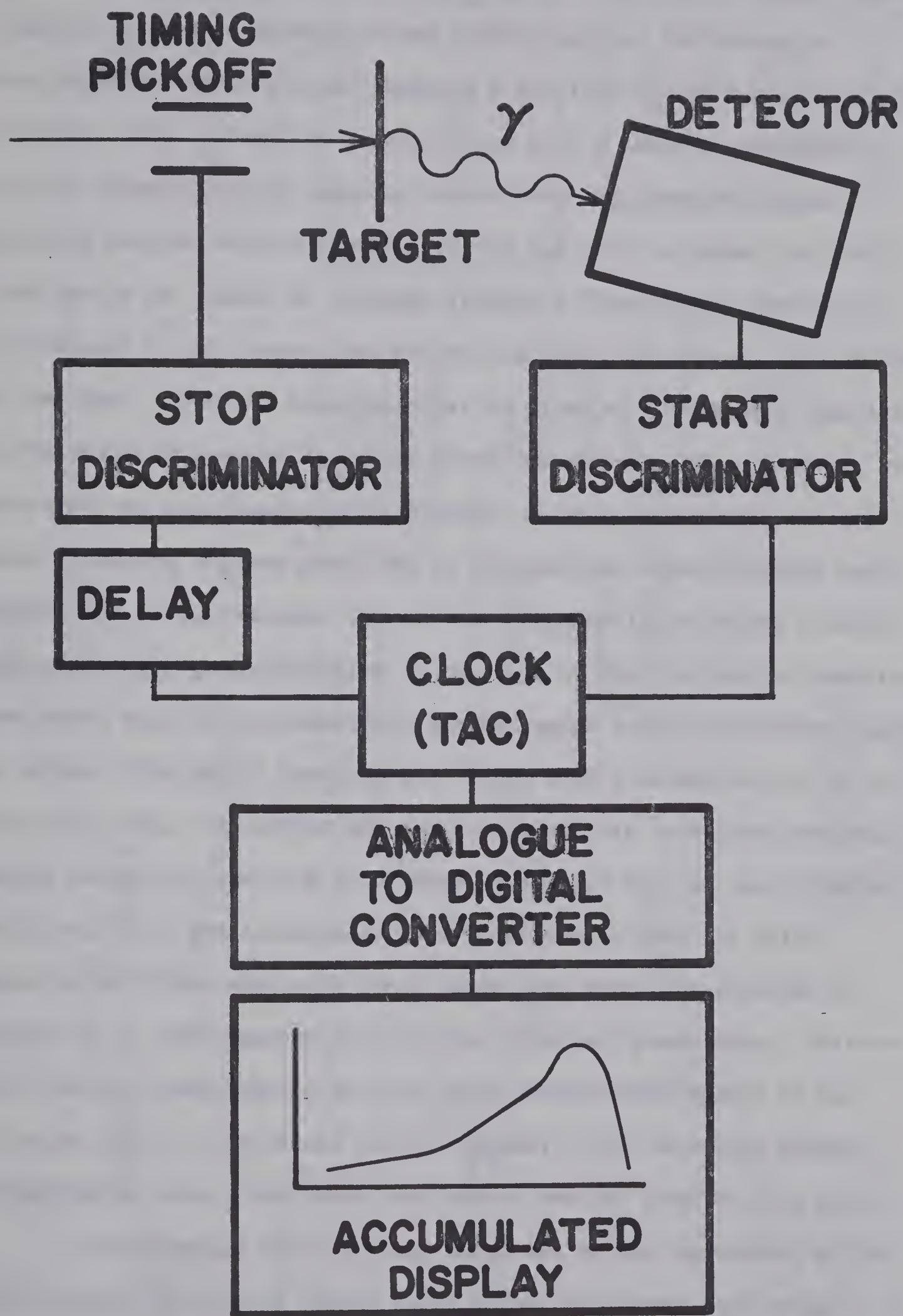
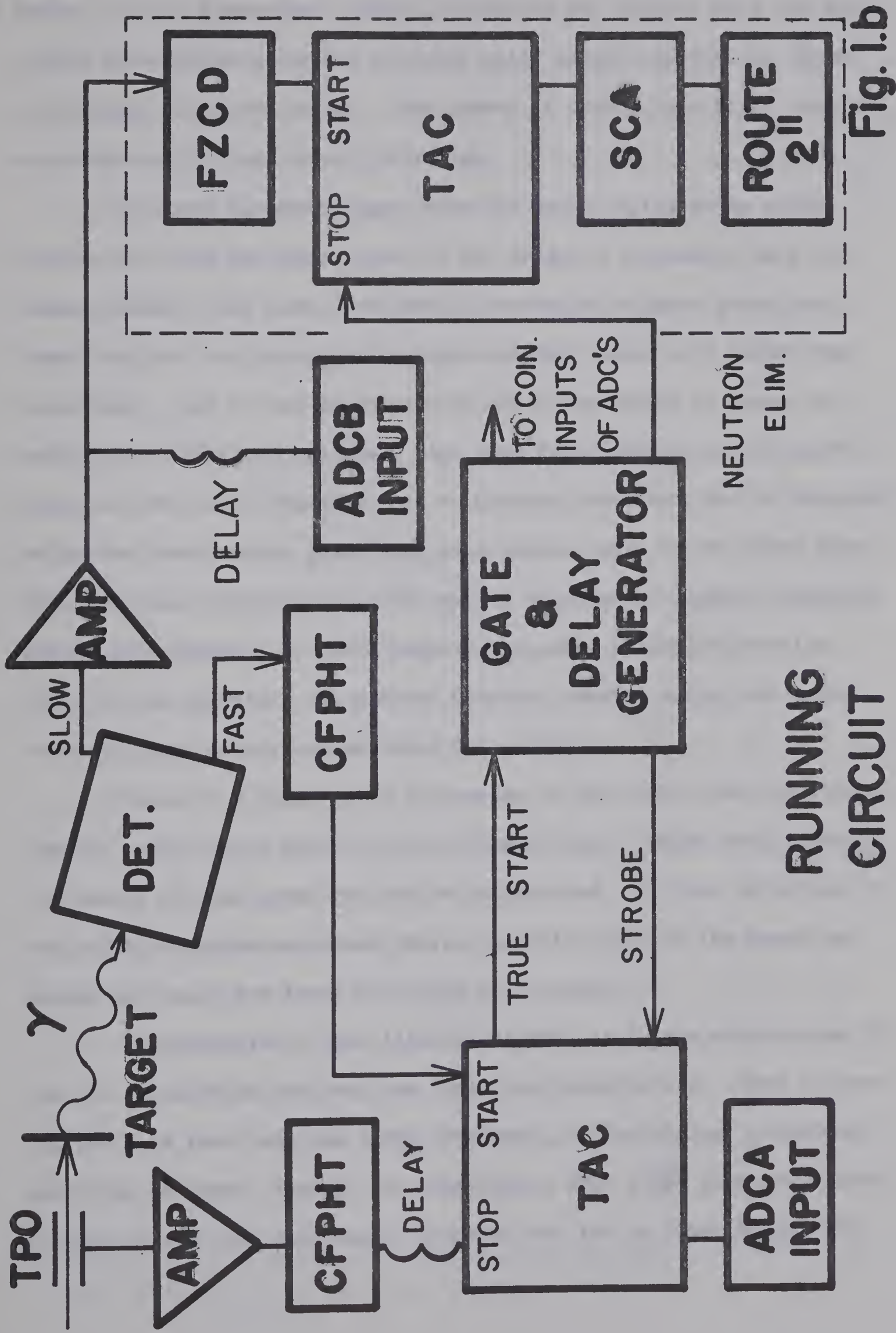


Fig. 1.a

computer can process by the Analogue to Digital Converter (ADC). The computer is used to accumulate and display many of the events in a decay spectrum whose precise shape is a function of the nuclear lifetime involved. This circuit is generally run with a neutron elimination circuit (figure 1.b) to separate events from the gamma rays from spurious neutron induced events. If one and only one gamma ray with a lifetime in the region of interest (termed a "long lived" gamma ray) is produced in the interaction of the beam with the target, the lifetime of the level producing this gamma ray is given by the natural logarithm of the slope of accumulated decay curve (cf. equation 2). However, when more than one such gamma ray is present, or when the identity of the level producing a given gamma ray is in question, identification techniques must be introduced. One useful technique is to record a Ge(Li) spectrum of the gamma radiation as the mode of data collection described just above does not differentiate between gamma rays of different energy or origin. The Ge(Li) spectrum will allow such a determination to be accurately made. A further sophistication of this technique involves gating the Ge(Li) spectrum with gamma rays which fall in the "lifetime tail" region of the accumulated decay spectrum so that the Ge(Li) spectrum will then show only those gamma rays which are produced in cascade (i.e. simultaneously) with the "lifetime" gamma rays. This can help identify gamma rays from long lived levels which appear in the "lifetime tail" of the decay curve. However, this technique suffers predominantly from a low count rate and so was not used in this study.

The technique that was used makes use of the dependence of the scintillation detector's output pulse height with gamma ray energy. It



is called the two dimensional (TWOD) collection and display mode and gives timing information along one axis and pulse height information (gamma ray energy) along the second. The number of counts in a given channel are displayed in the "third" dimension.

For each incoming signal from the detector, the pulse height information from the slow output of the detector is used to sort the timing signals into bins, the lower pulse height signals going into lower bins and the larger pulse height signals going into higher numbered bins. Due to the dependence of the pulse height on gamma ray energy, the higher energy gamma rays give decay spectra in the upper bins (and the lower bins too, due to Compton scattering in the detector) while the lower energy gamma rays give spectra only in the lower bins. With a correct calibration of the second analogue to digital converter (ADCB) (cf. figure 1.b) which handles the pulse height information input to the computer, the maximum Compton transfer energy and hence the full gamma energy can be found for each bin.

Thus, if a decay curve is present in the lower bins, by noting the bin number above which it is no longer found a rough indication of the energy of that gamma ray may be ascertained. By then referring to the Ge(Li) spectrum mentioned above, identification of the gamma ray energy and hence its level of origin may be made.

Determination of the lifetime is made as in the simple case if the bin in question has only one long lived contributor. When a given bin has more than one long lived component, an "unfolding" procedure must then be used. Assume, for simplicity, that these ^{γ_{ee}} gamma rays contribute to the time spectrum in a given bin, two of these being from

long lived states (one much longer (#1) than the other (#2)) and one (#3) which comes from a state whose lifetime is so short that the intrinsic resolution of the circuitry masks its gamma decay curve. To start the unfolding procedure the natural logarithm of the decay curve in the bin in question is plotted. An example is shown in figure 1.c.

The straight line at the left is due to the decay tail of the longest lived state (#1). This lifetime may be measured by applying equation 2.b to this line between the points A and B. Point B is chosen so that the contribution from the second longest lived state (#2) has fallen to a value which is less than the statistical error at that point. The antilogs are taken of the values on the broken line between B and C and these are then subtracted from the antilogs of the points B and E. When this is done the natural logarithm is taken of the new curve between B and E and this is plotted again. This new plot will have a straight line between F and D, the latter chosen so that the contribution from the system resolution (which masks the shortest state) has fallen to a value which is less than the statistical error at that point. Applying equation 2.b to this straight line gives the lifetime of the #2 gamma ray and the whole process is repeated to get the system resolution slope which in effect defines the shortest lifetime which can be measured. When the unfolding process is completed (in this case after two subtractions have been made), all that is left is a peak due to the system resolution called the "prompt" peak. It is the same prompt peak that would be displayed if there were only very short lifetime gamma rays being collected in a given bin, gamma rays with lifetimes so short that the slopes of their natural logarithmic plots are steeper than the slope due to the system resolution.

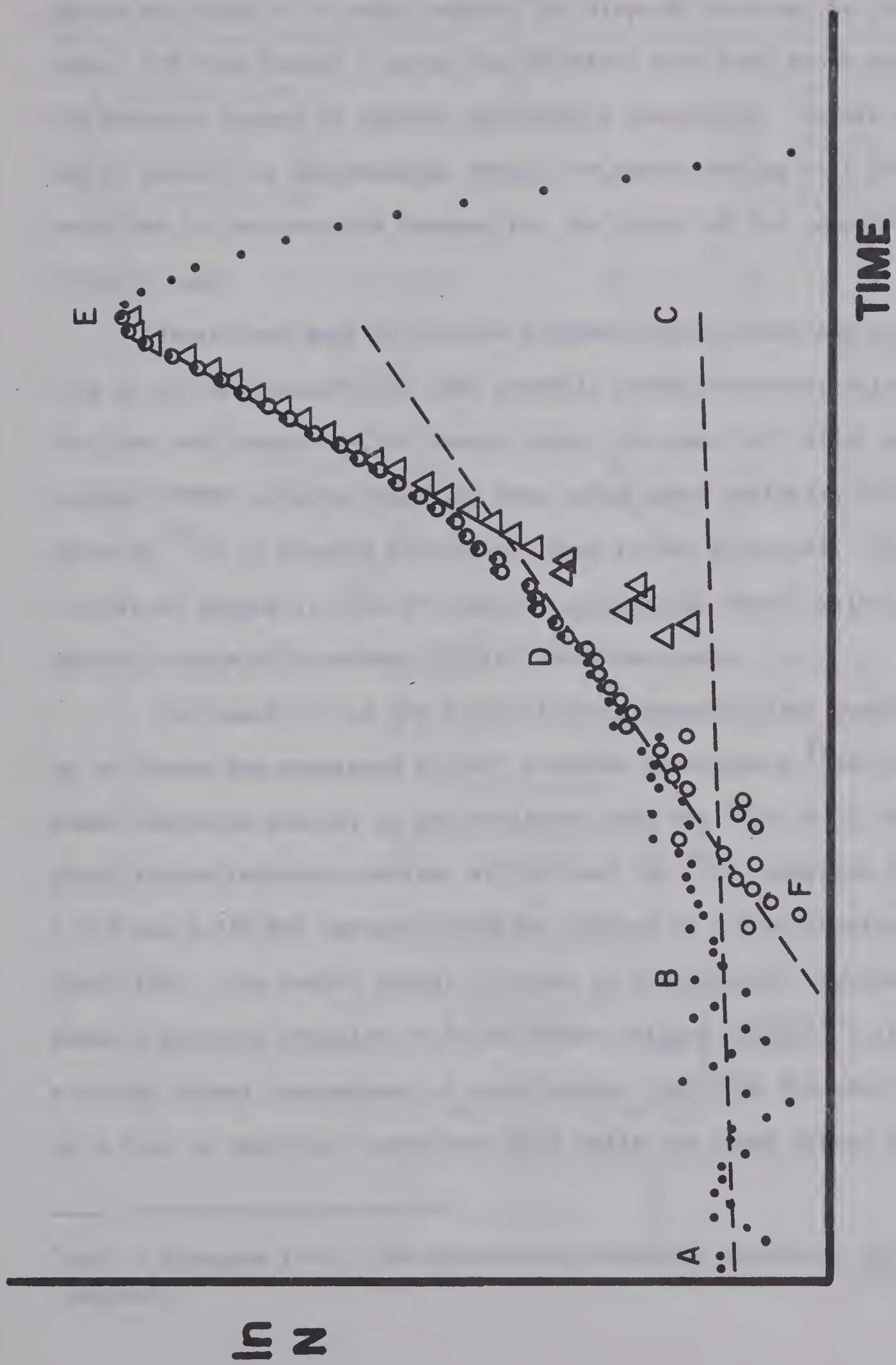


Fig. 1.c

From the above, it can be seen that if shorter lifetimes are to be measured the system resolution or prompt peak should be made as narrow as possible to avoid masking the slope of interest by the prompt peak. For this reason a great deal of effort went into bench testing of the detector system to improve the overall resolution. Before describing in detail the experimental setup, the bench testing will be described to indicate the reasons for the choice of the components actually used.

Tests were made of various photomultiplier tubes and scintillators to try and achieve the best possible timing characteristics. At the time work began on this thesis study, the best full width at half maximum (FWHM) achieved was ~210 psec using gamma radiation from the decay of ^{60}Co to provide coincident input to two detectors. The circuit of figure 2, with 1" diameter cylindrical Naton* scintillators mounted on RCA 8575 photomultiplier tubes was used.

In figure 2, the two scintillator-photomultiplier combinations to be tested are separated by 180° centered on either a ^{22}Na or ^{60}Co gamma radiation source, as the radiation from the ^{22}Na is 511 keV annihilation radiation emitted at 180° and the ^{60}Co radiation is 1.173 and 1.332 MeV cascade radiation emitted in a distribution peaked about 180° . The output signal from one of the detector combinations feeds a Constant Fraction of Pulse Height Trigger (CFPHT)²³ (to give a timing signal independent of pulse height) and then the start channel of a Time to Amplitude Converter (TAC) while the other signal goes into

*Nash & Thompson Ltd., Hook Rise South, Tolworth, Surbiton, Surrey, England.

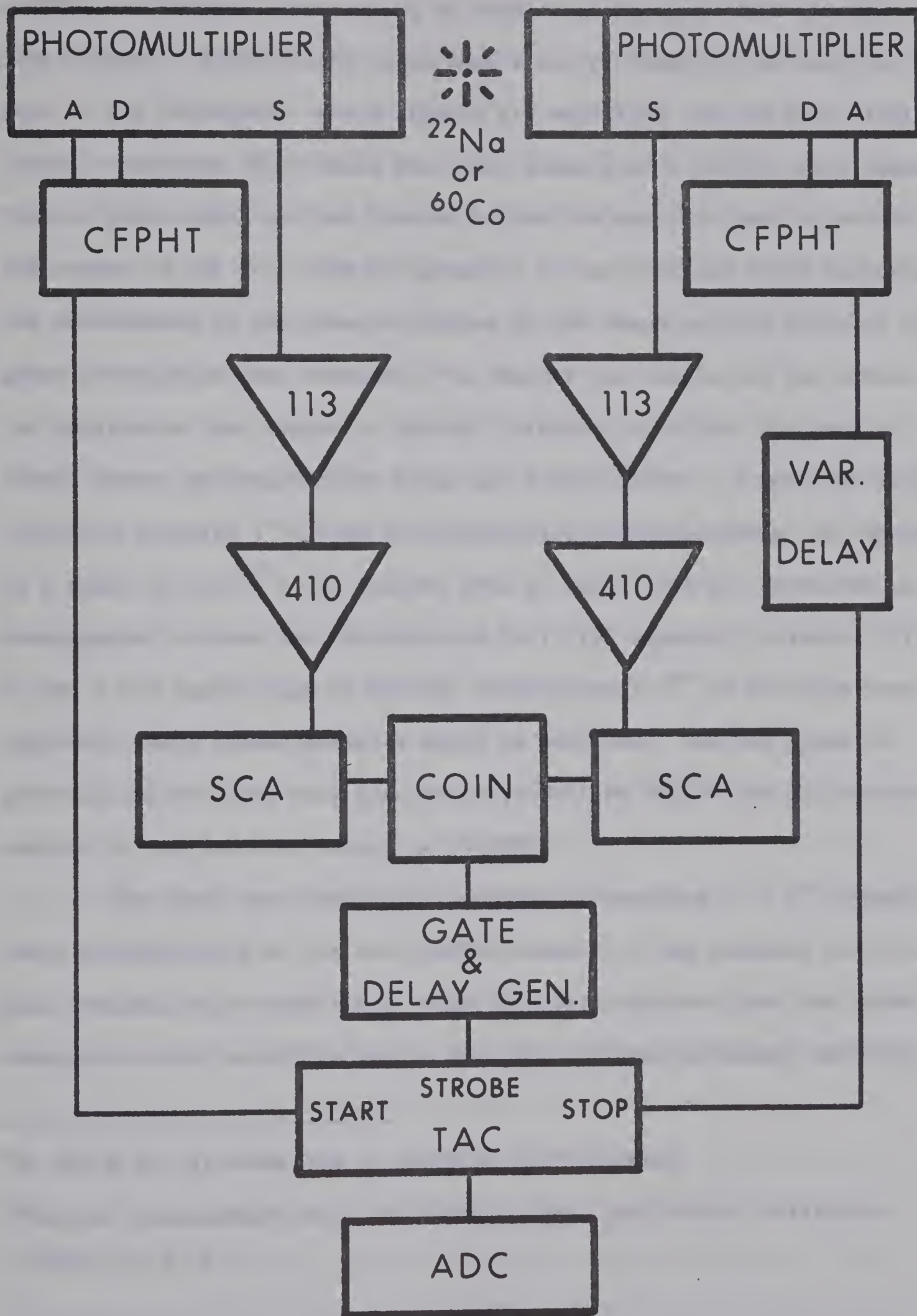


FIG. 2

a CFPHT, is delayed a set amount of time, and then goes into the TAC stop channel. Pulse height requirements are provided by the slow outputs of the detectors. These signals are amplified and fed into Single Channel Analysers (SCA) which pass only signals of a certain size range. Those signals which are not blocked by the SCA are then used to strobe the output of the TAC. The ADC presents a time spectrum which indicates the performance of the detector system by the shape and the width of the gamma coincidence peak produced. To improve the resolution and extend the applicable time region to shorter lifetimes an effort was made to obtain better photomultiplier tubes and scintillators. A new fast scintillation material 1¹-4⁴-bis (2-butyloctyloxy)-P-quaterphenyl as reported in a paper by Lynch²⁴) was ordered from E. Merck* and was dissolved in deoxygenated toluene and encapsulated in 1 1/2" diameter cylinders, 1/2, 1, and 1 1/2 inches high by Nuclear Enterprises Ltd** in the hope that improved timing characteristics might be achieved. Two new types of photomultiplier tubes were also tested, a Phillips XP1210 and an improved version of the RAC 8575 tube, the C31000D.

The first test conducted consisted of mounting 1" x 1" diameter Naton scintillators on two new C31000D tubes. It was expected that the peak obtained with these tubes would have been narrower than the peaks obtainable with the 8575's due to the much improved secondary emission

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**Nuclear Enterprises Inc., 935 Terminal Way, San Carlos, California, 94070, U. S. A.

ratio of the first dynode of the C31000D, but the ^{60}Co FWHM was only 210 psec at best, about the same figure attainable with the 8575's.

A Phillips XP1210 photomultiplier tube had been obtained from the manufacturer and the published risetime figures are better than those of the C31000D (C31000D 10% to 90% anode pulse risetime is 2.1×10^{-9} sec, XP1210 risetime is 1.2×10^{-9} sec). This tube was expected to improve the resolution. However, when one of the C31000D's was replaced by the XP1210 the FWHM increased to ~270 psec. Since the overall gains of the C31000D and XP1210 are about the same (10^7), a possible explanation for the absence of any timing improvement with the latter is that its photocathode efficiency is lower than the efficiency of the C31000D photocathode, i.e. the luminous sensitivity of the C31000D is 85 $\mu\text{A}/\text{lm}$ measured with a tungsten filament lamp at a color temperature of 2870°K and the XP1210 has a luminous sensitivity of 45 $\mu\text{A}/\text{lm}$ from a similar lamp at a color temperature of 2854°K . The spectral response of the XP1210 is S11 while the C31000D spectral response is very close to S8*. The corrected ratio of the luminous sensitivities and hence photocathode efficiencies of the C31000D to those of the XP1210 is 1.9.

In order to understand the probable cause of the lack of timing improvement with the C31000D over the 8575, the operation of the photomultiplier must be briefly noted. Consider a short pulse of light striking the photocathode. Electrons are emitted in a time distribution peaked about a central time value and these are accelerated and strike the first dynode causing multiple secondary emission. These electrons then strike the second dynode, again causing multiple secondary

*RCA Tube Handbook HB-3 Vol 3, 1950.

emission. This process is repeated along the dynode chain until a large output pulse is produced at the last dynode and the anode of the tube.

For optimum system resolution the time that should remain as constant as possible is the time between the gamma ray striking the scintillator and the photomultiplier tube output pulse peak time. The timing properties of the tube are highly dependent upon the number of photoelectrons produced in the first one or two stages of the tube in the following way. If many electrons are produced from the photocathode they will form a smooth curve in a time distribution with a well defined peak position. While the number of electrons arriving in a portion of this curve of time Δt may vary slightly from pulse to pulse, because of the large number of electrons, the curve shape stays pretty much the same, and the peak position (central time value) remains the same from pulse to pulse.

During the multiplication stages, each photoelectron will have its "descendents" spread in their own time distribution curves, but summed at the output pulse they still form the smooth curve with a constant peak timing from pulse to pulse.

However, with few electrons emitted from the photocathode, a smooth curve is not formed and the peak position will change from pulse to pulse, causing a substantial timing jitter and loss of resolution.

From the above, it can be seen that if the first stage or two of the tube can be made to have very high statistics in the electrons, better timing results, and while both the 8575 and C31000D's have good efficiency in the photocathodes, the latter tube endeavours to become a more precise timing instrument by using a high gain gallium-phosphide

first dynode. This should have the effect of presenting the second dynode with smoother pulse than in the 8575. However, since the tubes behaved similarly, it is assumed that this effect is not as significant between the first and second dynode as between the photocathode and first dynode. This is thought to be because the flight path (and hence straggling) is much greater between the photocathode and the first dynode than between the first and second dynode.

While this testing was going on, a few runs were made to determine the optimum operating voltages for the photomultiplier tubes. These will, of course, vary with gamma ray energy, scintillator type and from tube to tube, but for the 1" x 1" diameter Naton scintillator and a ^{60}Co source the tube available seemed to perform optimally at about 2300 V for the 8575's and about 2050 V for the C31000D's.

At this point in the bench testing procedure the quaterphenyl scintillators arrived from the manufacturer and testing of them began. The two smallest detectors were first mounted, the 1/2" x 1 1/2" diameter on an 8575 and the 1" x 1 1/2" on a C31000D photomultiplier tube. The quaterphenyl scintillators were expected to reduce the FWHM appreciably as Lynch's paper²⁴) had given light decay time constants for them, which were substantially less than those for Naton (60% less for the energy transfer mean time and 20% less for the light emitting decay mean lifetime). However, the best FWHM which could be obtained was 207 psec from ^{60}Co (figure 3). To find the reason for the lack of improvement in the FWHM, a pulse height study was carried out by mounting a NaI crystal, a Naton crystal, and a quaterphenyl scintillator successively on the same photomultiplier tube (8575) and observing the

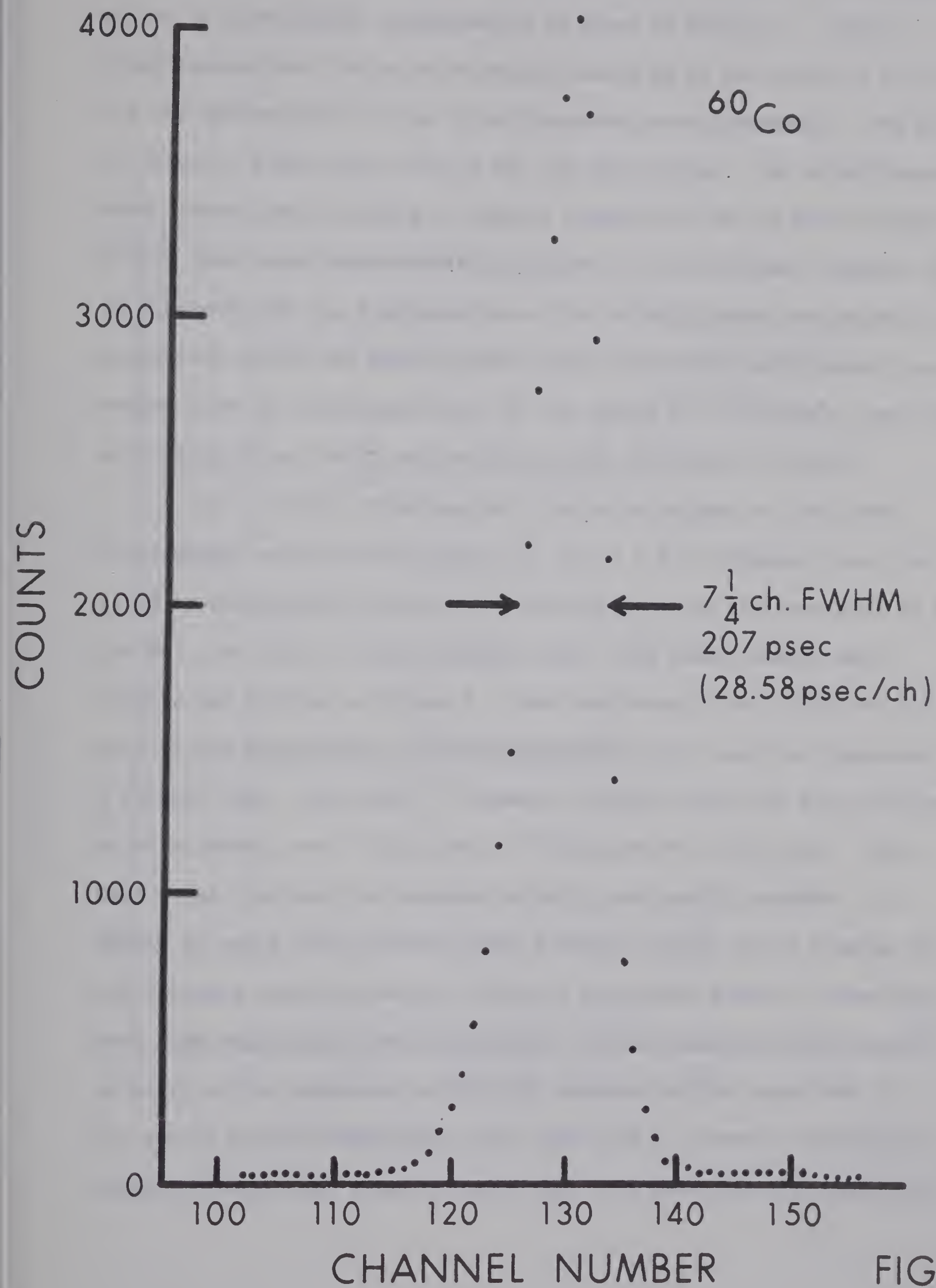


FIG. 3

pulse height by recording a ^{22}Na Compton spectrum from the slow channel. A plot of these pulse height spectra is given in figure 4. Lynch's paper states that the pulse strengths should be in the ratio of 9.0/1.4/2.1 for NaI/Naton/ $1^1,4^4$ -bis (2-butyloctyloxy)-P-quaterphenyl. The data of figure 4 gives 9.0/1.78/1.39 for the same ratios. The scintillators were constructed according to Lynch's formula and as the above results should have shown characteristics similar to his published figures, it is surmised that the deoxygenation of the scintillators was probably not sufficient during the manufacturing stage. The resultant reduced pulse height from the quaterphenyl may be the reason for the FWHM's from these scintillators not being smaller than those from Naton crystals.

As a further investigation, the pulse heights of the three quaterphenyl scintillators ((1/2", 1", 2") x 1 1/2" diameter) were compared by successively mounting the scintillators on the same 8575 as in the NaI v.s. Naton v.s quaterphenyl test. The lower Compton edge results are plotted in figure 5. From the plots it was noted how the size of the scintillator affected the pulse height and this suggested a further test. Six small 1" diameter cylinders were cut from the same block of Naton, two 1" high, two 1/2" high and two 1/4" high. These were highly polished and mounted in MgO lined plastic holders. (It should be noted that polished Naton surfaces should not be touched with any cleaning substance except absorbent cotton and plastic cleaner and even then very lightly and sparingly.) The scintillators were mounted in pairs of the same size on RCA 8575 photomultiplier tubes and the circuit of figure 2 was again used. The 1" x 1" diameter Naton scintillators were first tried and with 2200 V on each tube and 20% windows

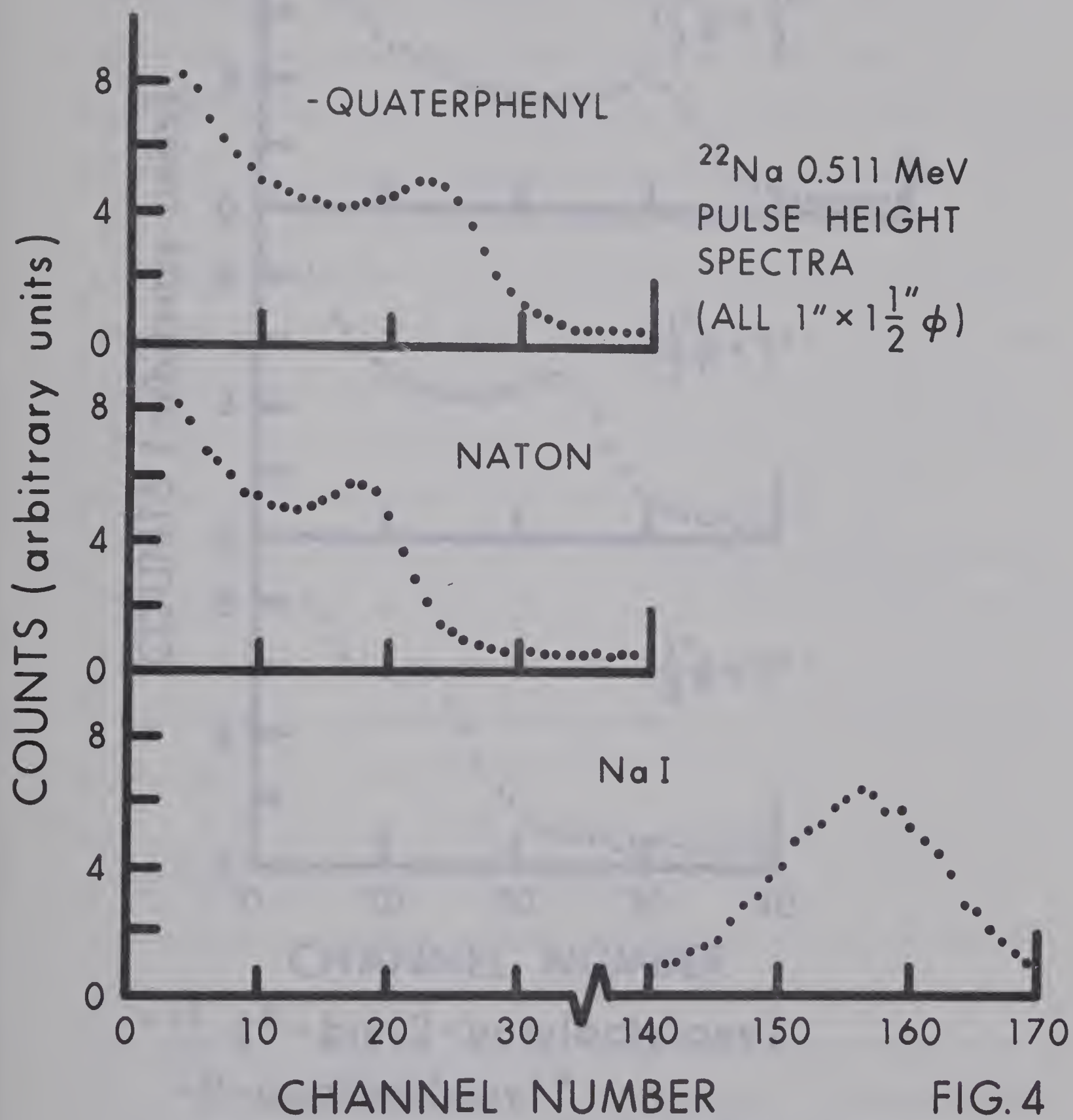
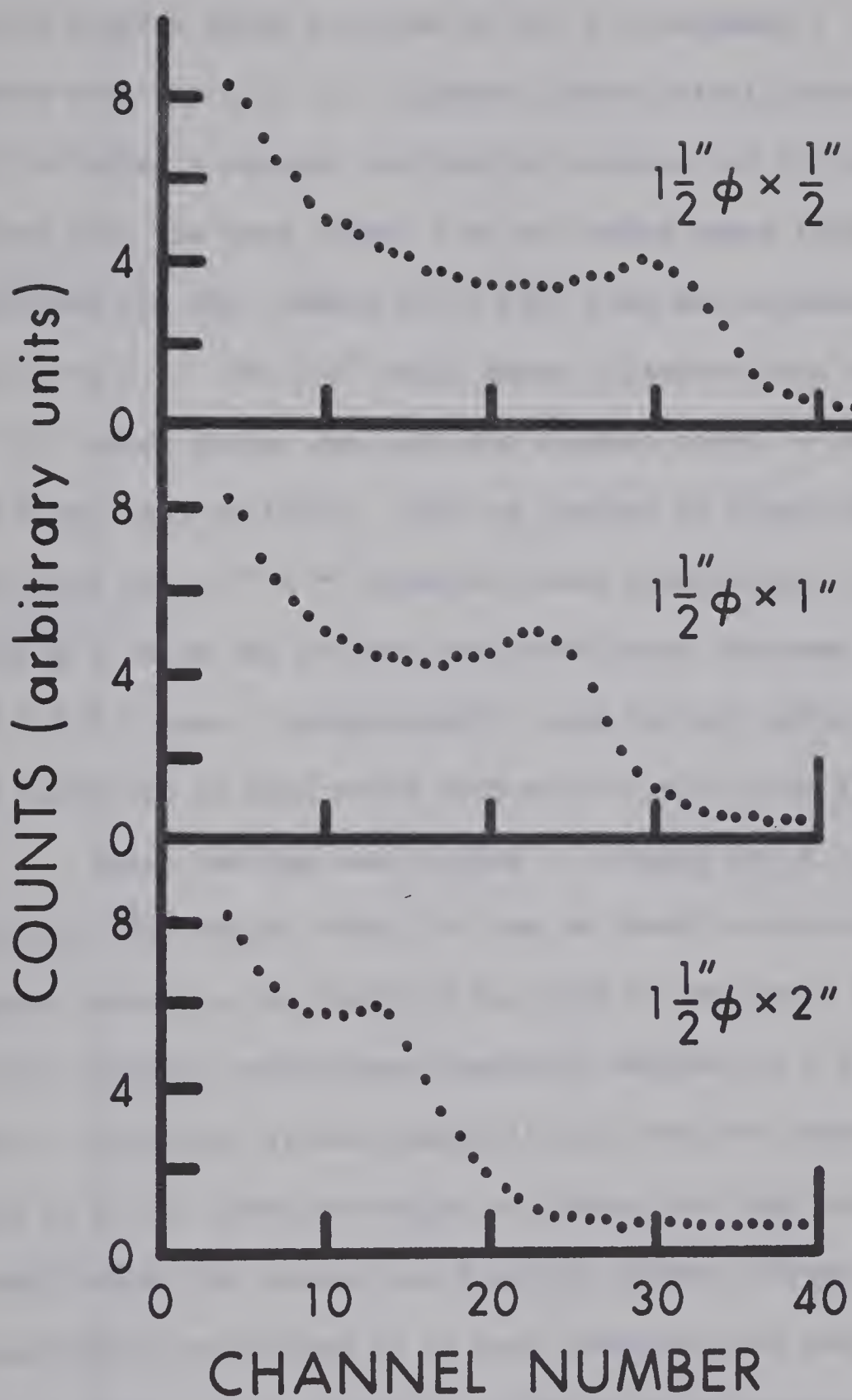


FIG. 4



"1¹, 4⁴ -bis(2-butyloctyloxy)
-P-quaterphenyl"

^{22}Na 0.511 MeV COMPTON EDGE

FIG. 5

on the SCA's an optimum FWHM of 193.6 ± 8.1 psec was obtained. This is plotted in figure 6.a. (These FWHM were fitted using a new version of the program Tepel provided by Mr. D. Orachesky.) With the same window size the $1/2'' \times 1''$ diameter Naton scintillators were mounted, and following a careful calibration carried out by inserting known delays into the stop signal line and using peaks thus created to calibrate the ADC, FWHM of 165.0 ± 6.9 psec was achieved. This is plotted in figure 6.b. The $1/2''$ thick Naton cylinders were removed and replaced by $1/4''$ thick pieces and with the windows reset to 20%, a FWHM of 162.9 ± 6.8 psec was achieved. This is plotted in figure 6.c. The measurement with the $1/2'' \times 1''$ diameter Naton scintillator was repeated a week later as a check and at that time FWHM under the same conditions was 170.4 ± 5.1 psec. Unfortunately, time did not permit further study of the variation of peak width with scintillator size at this time.

Bench testing was stopped to prepare for a lifetime measurement run on a ^{45}Sc target using the Van de Graaff accelerator. The detection system chosen on the basis of the data of the bench testing was a $1'' \times 1\ 1/2''$ diameter quaterphenyl detector mounted on a 8575 photomultiplier tube. Initially, it was placed 16 cm. from the target, but was moved back to 95 cm. from the target to reduce the high counting rate, when a very thick ^{45}Sc target was inserted midway through the run. This scintillator was chosen as it best combined fast timing characteristics with a large active volume to maximize the efficiency of the experiment. This detection system was used in the TWOD data collection mode described above. For this experiment (and others) Dr. W.K. Dawson of the University of Alberta has provided a flexible TWOD program with variable

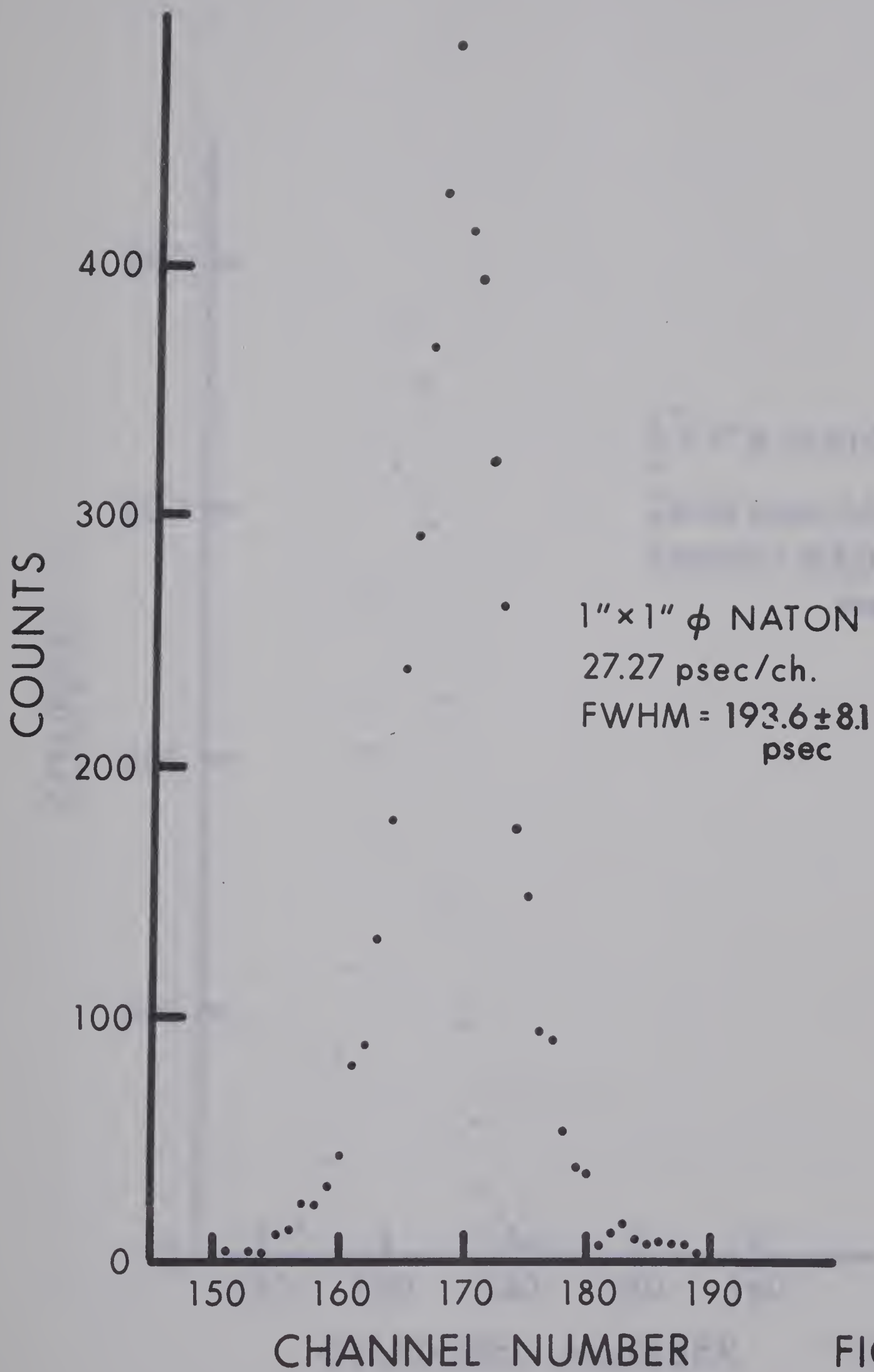


FIG. 6a

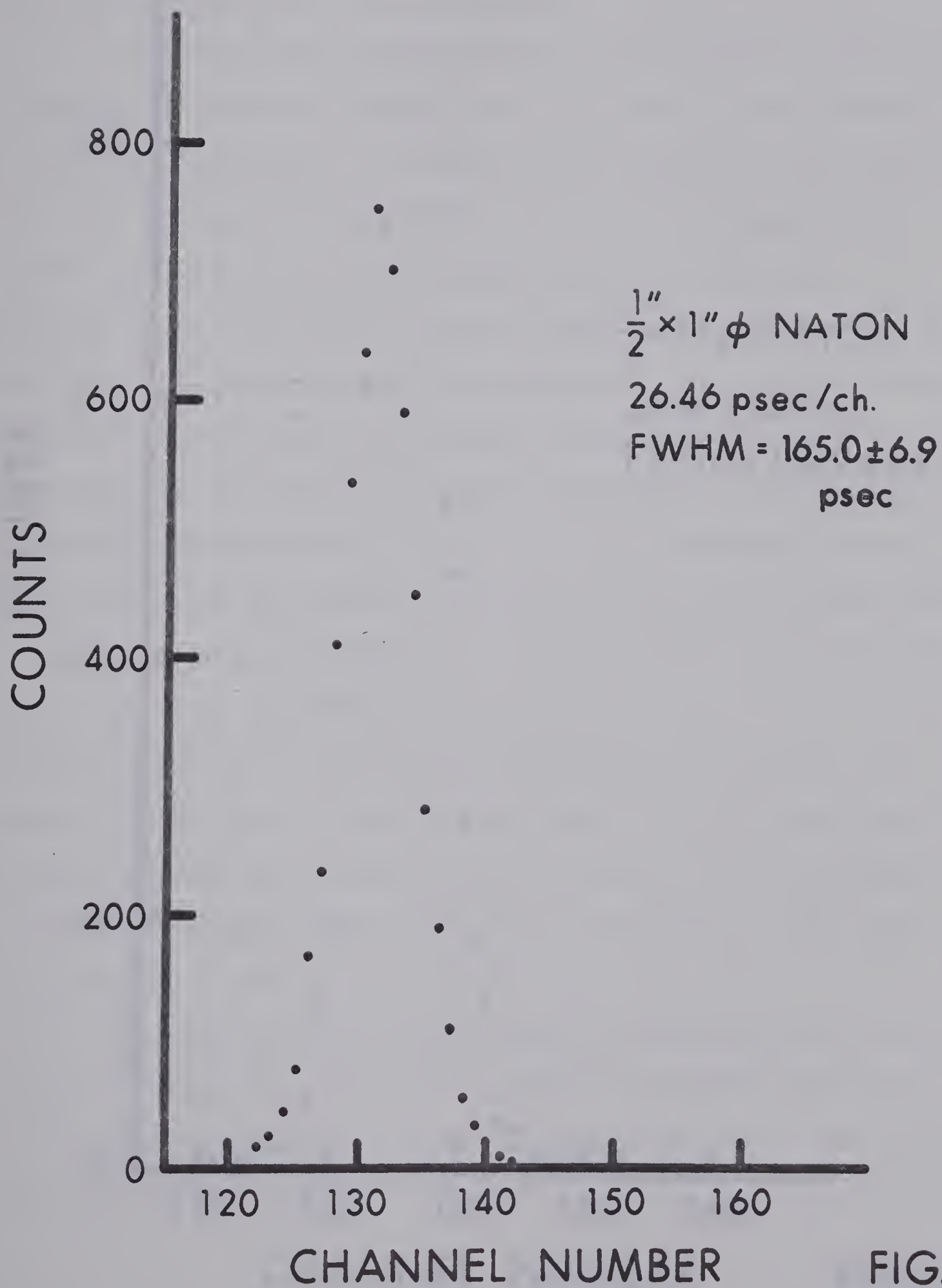


FIG. 6b

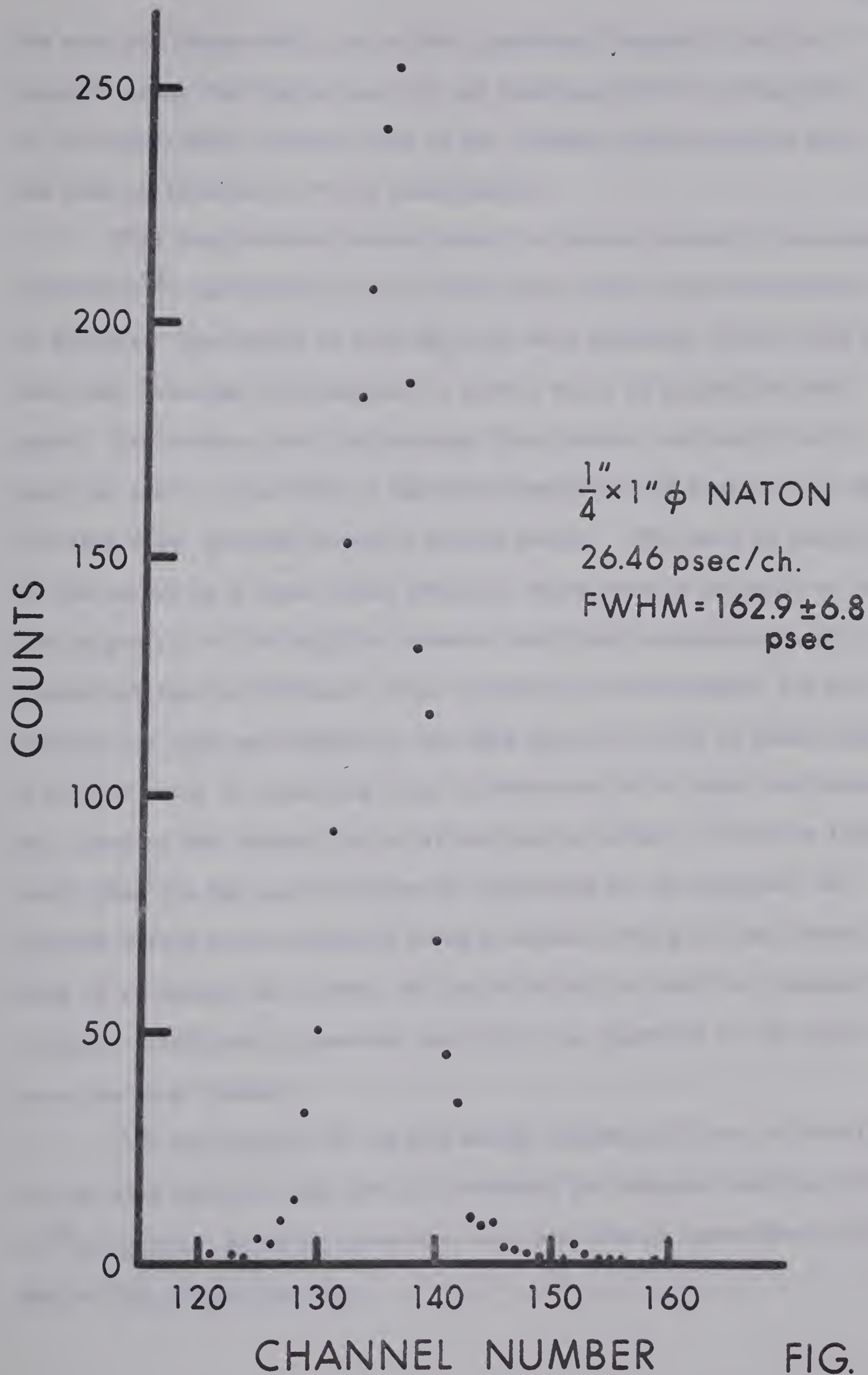


FIG. 6c

bin size and number which ran on the laboratory Honeywell DDP-516 computer using the display unit of the laboratory SDS 920 computer. In this experiment, sixteen bins of 256 channels each were used with the time calibration of 74.85 psec/channel.

This time calibration was done by a method basically developed by Hatcher²⁵) and modified by the laboratory staff at the University of Alberta. The output of a 10 MHz₂ sine wave generator is fed into a fast zero crossing discriminator to give a train of pulses 100 nsec apart. Two outputs from the crossing discriminator are used, one to feed the start of the Time to Amplitude Converter and the other to feed the stop after passing through a gating device. The gate is controlled by the output of a pulse train generator whose rate is adjusted so that the output of the TAC will be pulses of amplitude corresponding to a random multiple of 100 nsec. When collected by the computer and displayed, the time per channel of the ADCA spectrum could be ascertained. A problem arose in producing finer calibrations as at least two peaks must show on the display for a calibration to be made. With the time scale that the TAC was to be run on this could not be achieved, so several delays were calibrated using a suitable TAC scale and these were used to calibrate the correct TAC scale noting the shift in channels of a single artificially generated peak with the insertion of the delay into the stop channel.

The calibration of the bin energy (essentially the calibration of the ADCB spectrum) was done by recording the Compton spectrum from a ²²Na source. Assuming linearity, each bin covered approximately 250 keV of the energy spectrum.

For each run an ungated Ge(Li) spectrum of the gamma radiation was also recorded with a 30 cc. active volume Ge(Li) detector* placed 30.4 cm. from the target. After taking data collection runs at increasing bombarding energy steps, a comparison of the Ge(Li) gamma spectrum taken at a given energy with the spectrum taken at the immediately preceding energy step helped to identify gamma rays by defining the energy at which they are first produced and hence their nuclear level of origin. This technique is of particular use when dealing with a reaction with a negative Q value, i.e. a reaction which requires a positive energy input if it is to go at all. The reaction studied in this experiment, $^{45}\text{Sc}(p,n)^{45}\text{Ti}^*$ is such a reaction with a Q value of -2.841 MeV. This reaction, suggested as a candidate for such a study by Dr. K.V.K. Iyengar, involves bombarding ^{45}Sc with protons and ignoring the emitted neutrons while observing the decay of the excited states of ^{45}Ti which are formed. The competing reaction $^{45}\text{Sc}(p,p)^{45}\text{Sc}^*$ cannot be prevented and must be accounted for in all results such as lifetime and Ge(Li) spectra. Fortunately, the competing reaction $^{45}\text{Sc}(p,\alpha)^{42}\text{Ca}$ was not too prominent a reaction due to the high Coulomb barrier which the ^{42}Ca presents to the α cluster. Other competing reactions from protons on ^{45}Sc require input energies in excess of 10 MeV and so are not considered in this work.

The targets were generally the so called "goop" or "gloop" targets which consist of a mixture of polyurethane dissolved in benzene into which the finely powdered target material is mixed. The matrix

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holding the target material is thus composed of only carbon, oxygen and hydrogen, none of which have levels with lifetimes that would interfere with any experiments being carried out. This brings up an important experimental point, that the beam should not be able to hit anything except the target or pure gold as other materials might present "long lived" gamma rays (e.g. tantalum 10.6 ns, 482 keV level) which would interfere with the gammas of interest. The target materials were thus "glooped" onto 0.020" gold backings which were in turn mounted on a 2 1/2" diameter disc (with spring clips on one face to hold the gold-backed target). This disc fitted over the end of the beam tube and formed a butt vacuum seal with that end and was also secured in place by a plastic retaining ring. Unfortunately, the disc was constructed of tantalum as it was designed for another experiment which did not involve lifetime studies and hence did not require this same precaution. To solve this problem a gold annular shield was constructed to fit inside the beam tube completely covering the interior end except for a 1/2" diameter opening exposing the target to the beam.

A stub extending from the tantalum disc through a slot on the retaining ring's inner face formed a connection for the Faraday cup and cooling of the targets was provided by air blown against the back of the disc. On occasion when a thick metal target (which would not let the beam strike the tantalum) was used, the target was mounted without gold backing directly on the disc to provide better cooling.

The run was begun with a ^{10}B target instead of ^{45}Sc to check the operation of the circuit. The data of figure 7 (a natural logarithm plot) was taken with a charge collection of 3972 μc in just over forty-

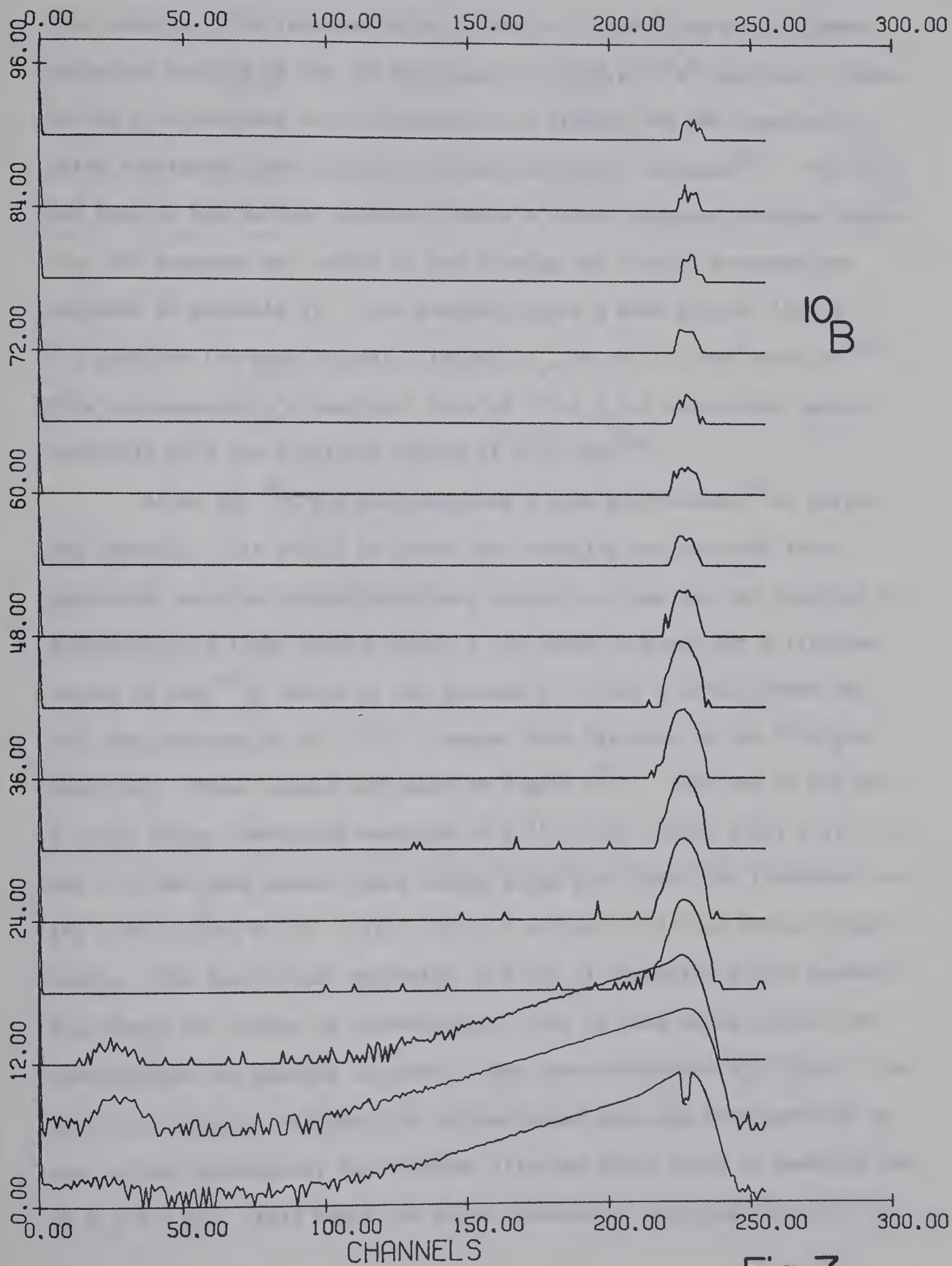


Fig. 7

five minutes. The lifetime tails in bins 0, 1 and 2 are due to gamma radiation emitted by the 717 keV level of $^{10}\text{B}(\text{p},\text{p}')^{10}\text{B}^*$ reaction. These curves were analysed on the University of Alberta IBM 360 computer by using a weighted least squares calculation due to Ferguson²⁶). The work was done on the Nuclear Research Centre's remote computer terminal using the APL language and copies of the fitting and display programs are included in Appendix III. The analysis gives a mean life of 1026.3 ± 8.6 psec for the mean nuclear lifetime, τ_m , of the 717 KeV level of ^{10}B . This corresponds to a mean half life of 711.6 ± 6.0 psec which agrees favorably with the published figure of 0.71 nsec ¹⁹).

After the ^{10}B run was completed a thin gold-backed ^{45}Sc target was inserted. (It should be noted that changing targets with this apparatus could be accomplished very quickly as time was not required to depressurize a large vacuum vessel.) In order to carry out a lifetime survey of the ^{45}Ti levels it was necessary to have a level scheme and this was provided by Dr. K.V.K. Iyengar from his work on the $^{45}\text{Sc}(\text{p},\text{n})$ reactions. These levels are shown on figure 8²⁶). With the -2.841 MeV Q value proton bombarding energies of 3.15, 3.35, 3.955, 4.25, 4.3, 4.45 and 4.55 MeV were chosen, each energy being just above the threshold for the 0.40, 0.330, 0.740, 1.225, 1.350, 1.465 and 1.515 MeV levels respectively. The data is not presented in order of ascending proton bombarding energy but rather in chronological order as this makes evident the unsteadiness and general decrease of the beam resolution with time. The beam was initially adjusted for optimum pulse size and sharpness and at the optimum resolution, the shortest lifetime which could be measured was 92.5 ± 5.1 psec (this being the value obtained by applying equation 2.b

Kashy and Conlon Via $^{46}\text{Ti}(\text{p,d})$ ^{45}Ti Reaction $E_{\alpha}(\text{MeV})$ $\ln$ J^{π}	L'écuyer and St-pierre Via $^{46}\text{Ti}(\text{}^3\text{He},\alpha)$ ^{45}Ti Reaction $E_{\alpha}(\text{MeV})$ $\ln$ J^{π}	Lutz and Bohn Via $^{46}\text{Ti}(\text{}^3\text{He},\alpha)$ ^{45}Ti Reaction $E_{\alpha}(\text{MeV})$ $\ln$ J^{π}	McCallum et al. Via $^{45}\text{Sc}(\text{p,n})$ ^{45}Ti Reaction $E_{\alpha}(\text{MeV})$	Jett et al. Via $^{45}\text{Sc}(\text{p,n}\gamma)$ ^{45}Ti Reaction $E_{\alpha}(\text{kev})$	Present experiment Via $^{45}\text{Sc}(\text{p,n})$ ^{45}Ti Reaction $E_{\alpha}(\text{MeV})$
0 3 $7/2^{-}$	0 3 $7/2^{-}$	0 3 $7/2^{-}$	0	0	0
0.33 2 $3/2^{+}$	0.33 2 $7/2^{+}$	0.33 2 $3/2^{+}$	0.025 $\pm$ 0.004	36.63	0.040 $\pm$ 0.010
	0.79		0.740 $\pm$ 0.008	40.15	0.330 $\pm$ 0.010
			(0.786 $\pm$ 0.010)	329.45	0.740 $\pm$ 0.010
			(0.859 $\pm$ 0.010)	743.9	
1.37			1.232 $\pm$ 0.012	1227.11	1.225 $\pm$ 0.010
1.49		1.49 $1/2$	1.332 $\pm$ 0.012		1.350 $\pm$ 0.010
1.58	1.38		1.445 $\pm$ 0.012		1.465 $\pm$ 0.010
1.79	3.10				1.515 $\pm$ 0.010
	4.79				
	5.78				1.800 $\pm$ 0.015
					1.880 $\pm$ 0.015
					2.020 $\pm$ 0.015

Fig. 8

to the left hand side of the prompt peak). The peaks in the upper bins of figure 9a ($E_p = 3.955$ MeV) were taken as the prompt peaks and application of the least square fitting program gave their inverse slopes (lifetime) as in table III.

TABLE III

Bin 11	94.2 ± 5.2 psec
Bin 12	107.7 ± 5.6 psec
Bin 13	92.5 ± 5.1 psec
Bin 14	98.2 ± 6.2 psec

After three runs had been taken with the thin target ($E_p = 3.955, 4.3, 4.45$ MeV; figures 9a, 9b, and 9c respectively), a very thick (0.005") scandium foil was inserted. The detector was moved back and another run was taken at 4.45 MeV, shown in figure 9d. The data taken at $E_p = 4.55$ MeV is shown in figure 9e, 4.25 MeV in figure 9f, 3.35 MeV in figure 9g and 3.15 MeV in figure 9h. The actual data taking was comparatively uneventful apart from slight trouble with the beam conditions drifting (cf. figure 9b).

Due to this beam drifting the prompt peaks taken initially could not be used throughout the run and so analysis of the runs could not proceed by the centroid shift or by the conventional method given in section III. As the prompt peak could not be subtracted, the shortest lifetimes are uncertain as they could not be separated from the prompt peak contribution. The method of analysis used consisted of visually scanning the natural logarithmic plots of the data to find any peaks

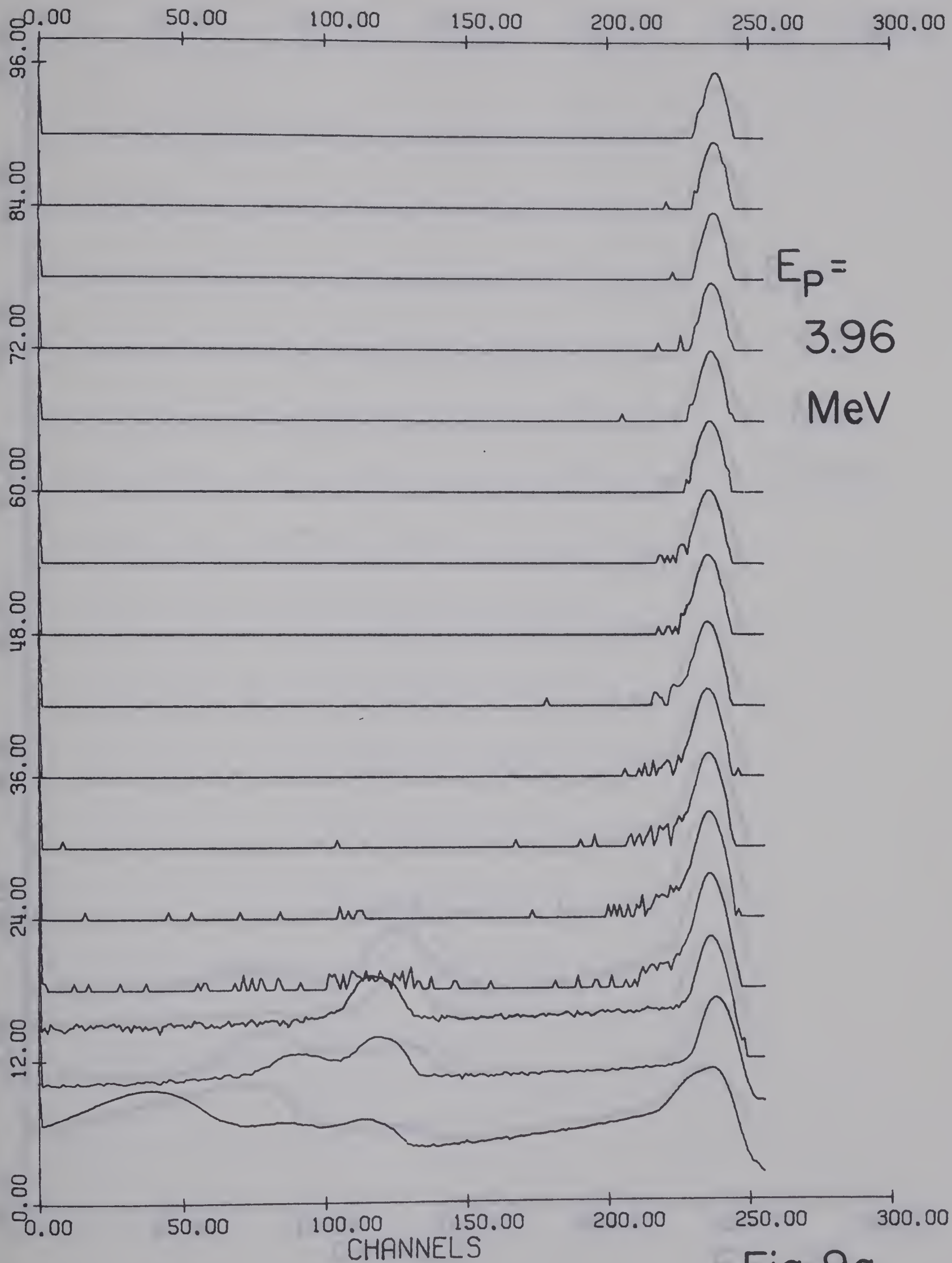


Fig. 9.a

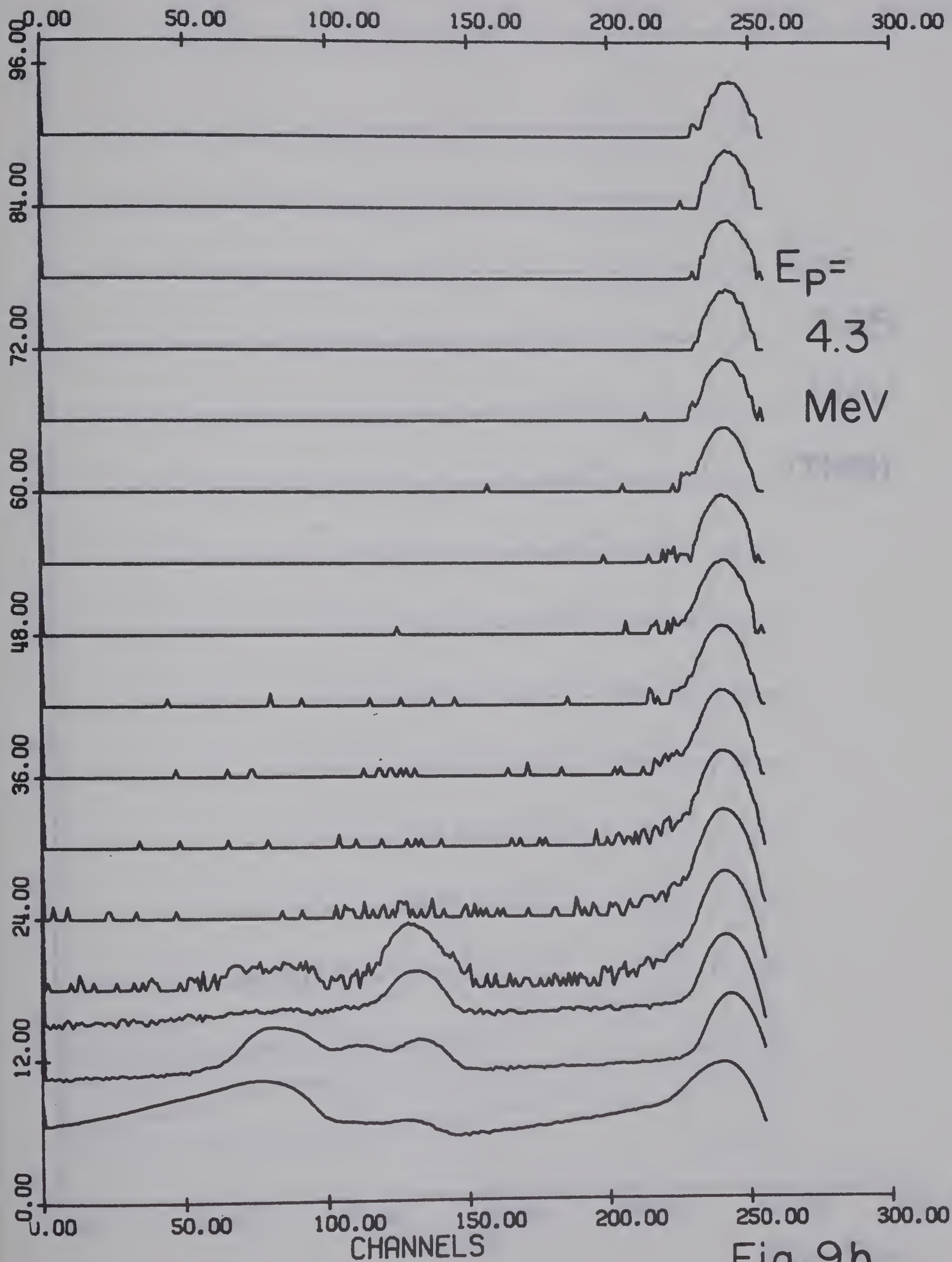


Fig. 9.b

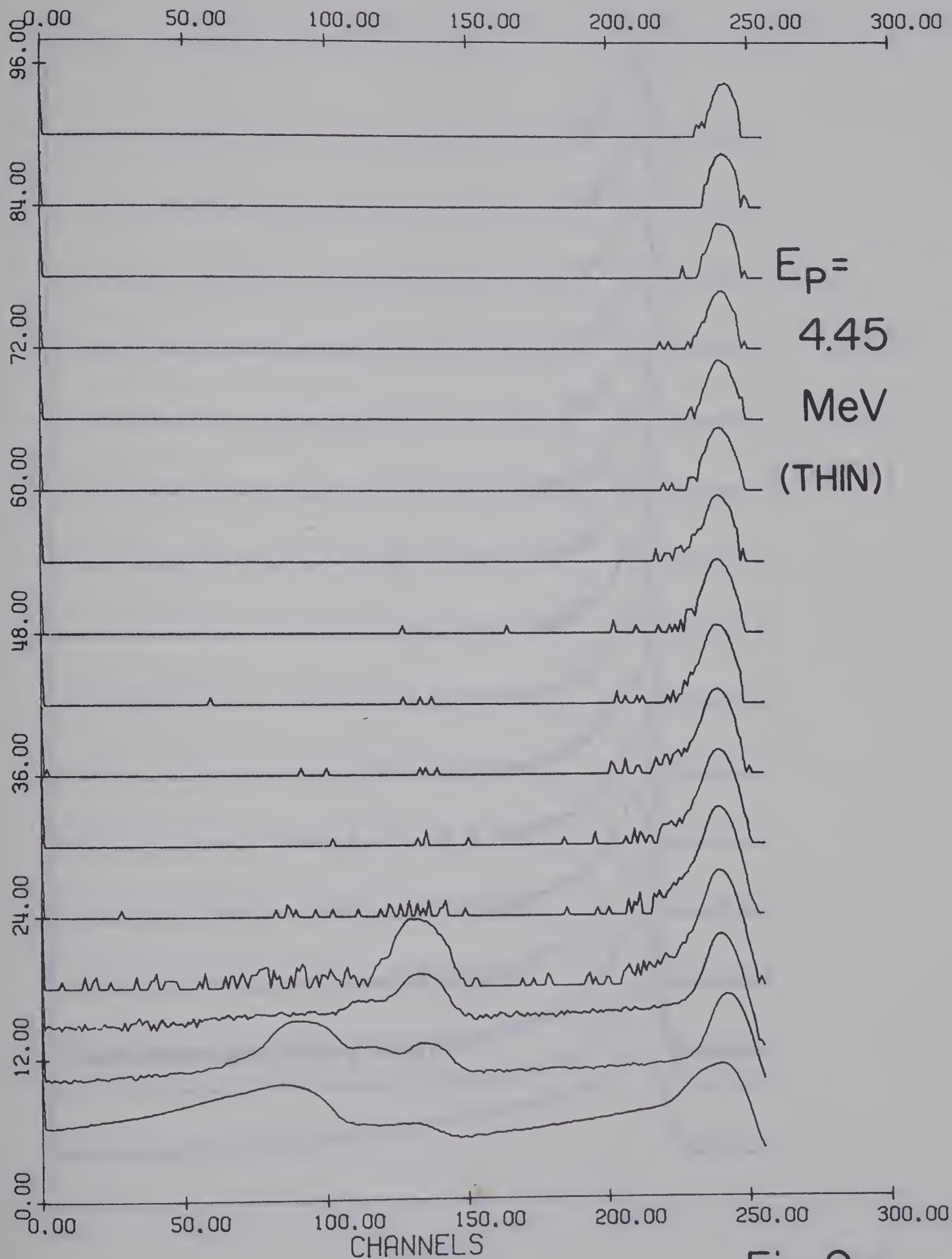


Fig. 9.c

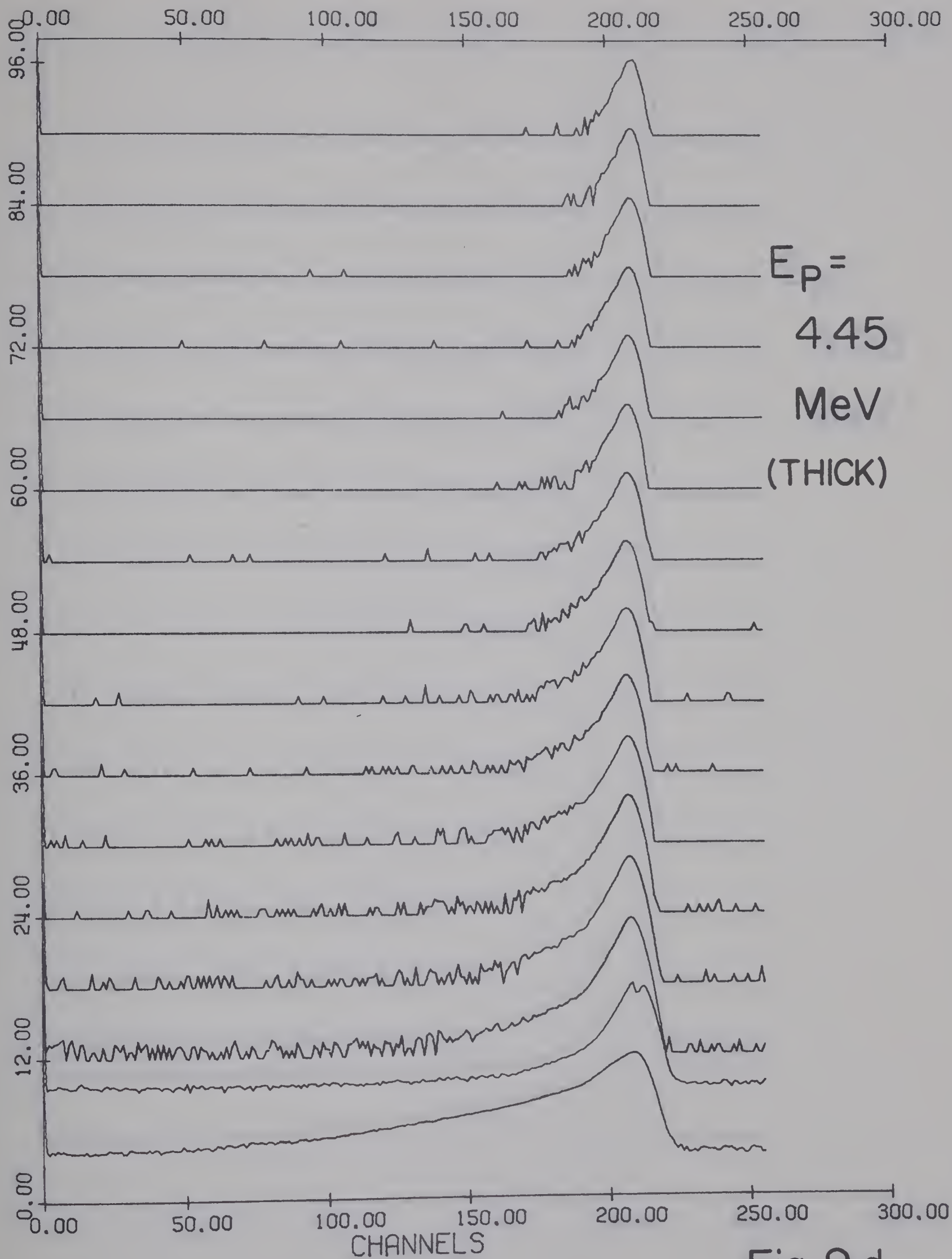


Fig. 9.d

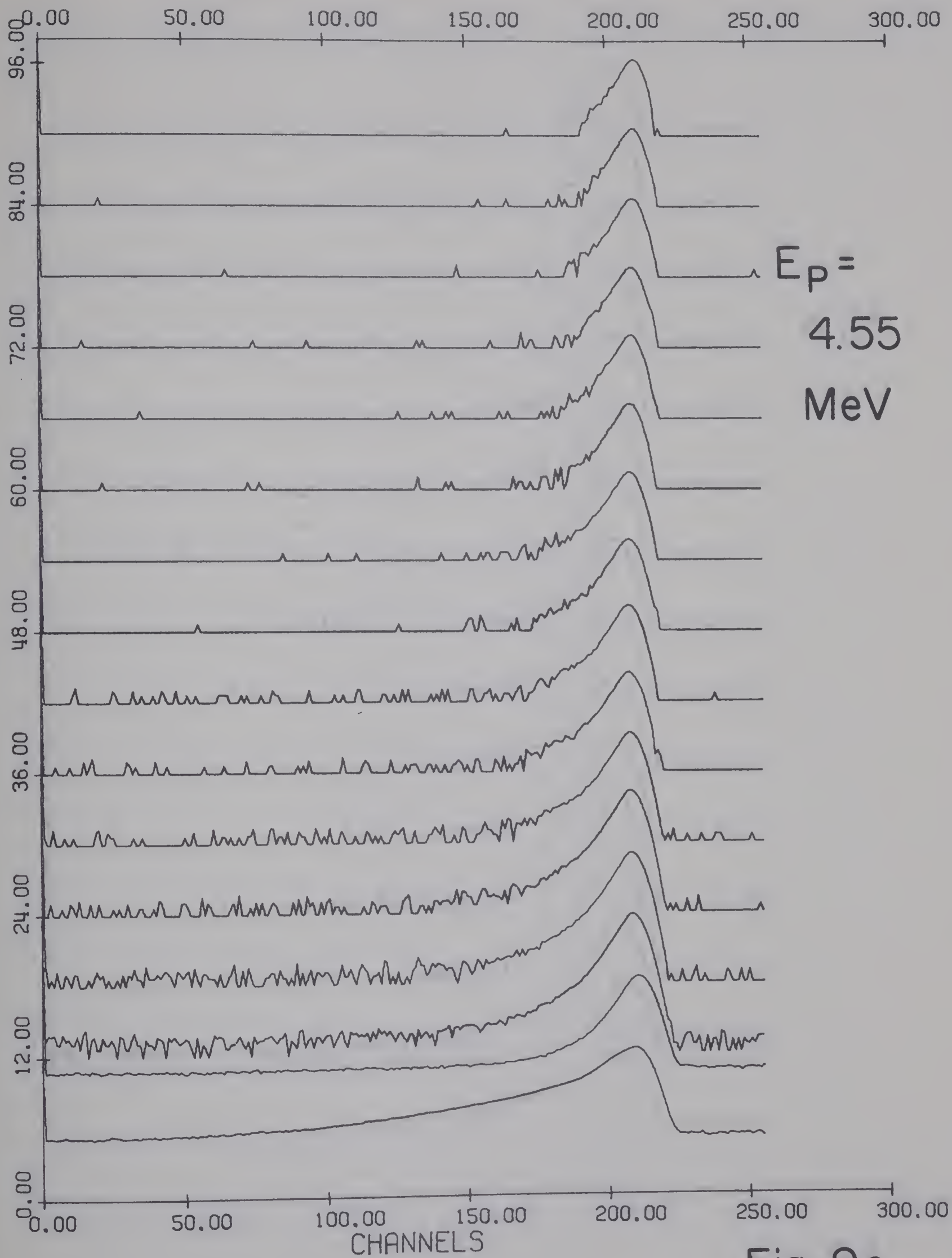


Fig. 9.e

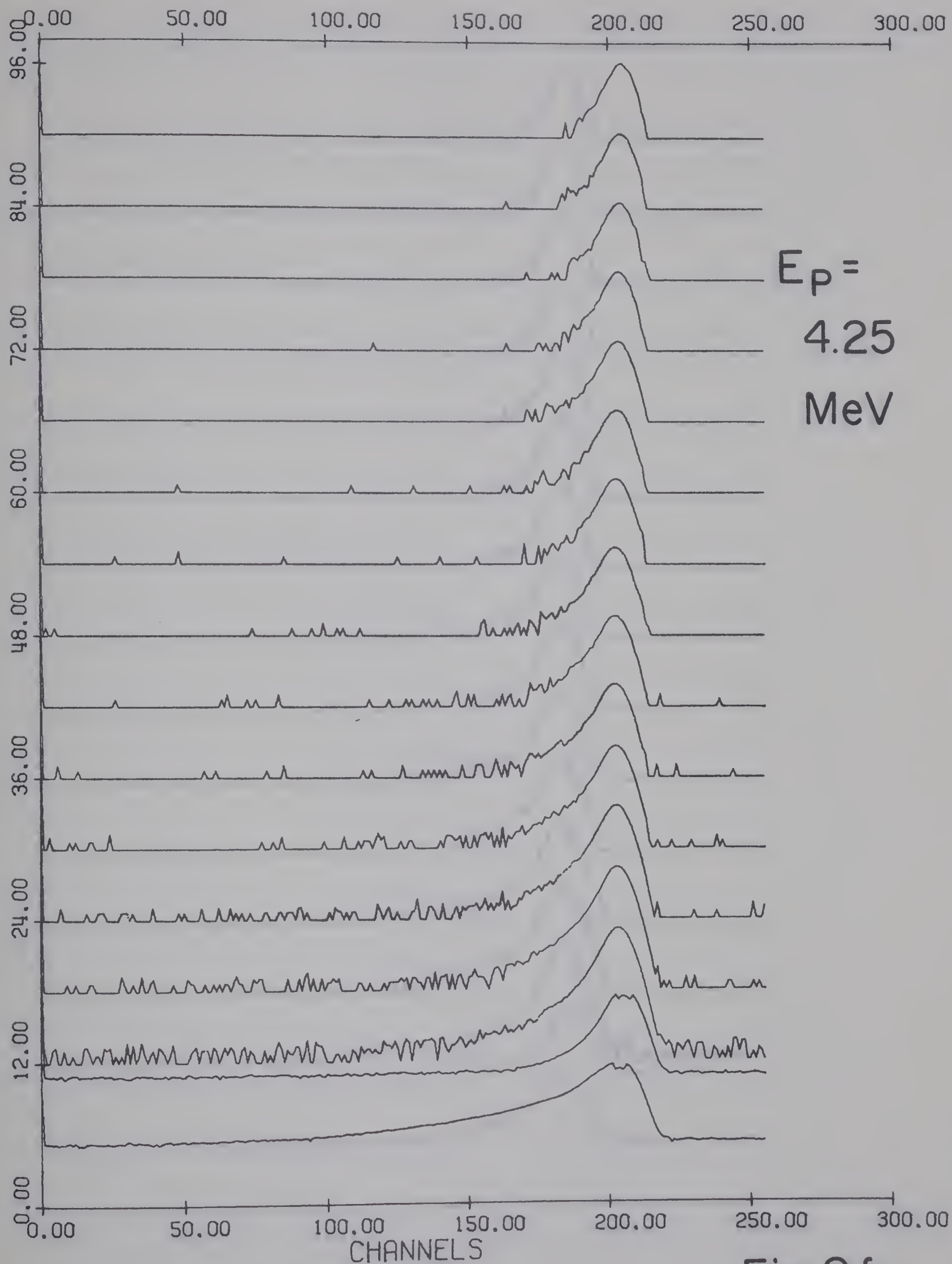


Fig. 9.f

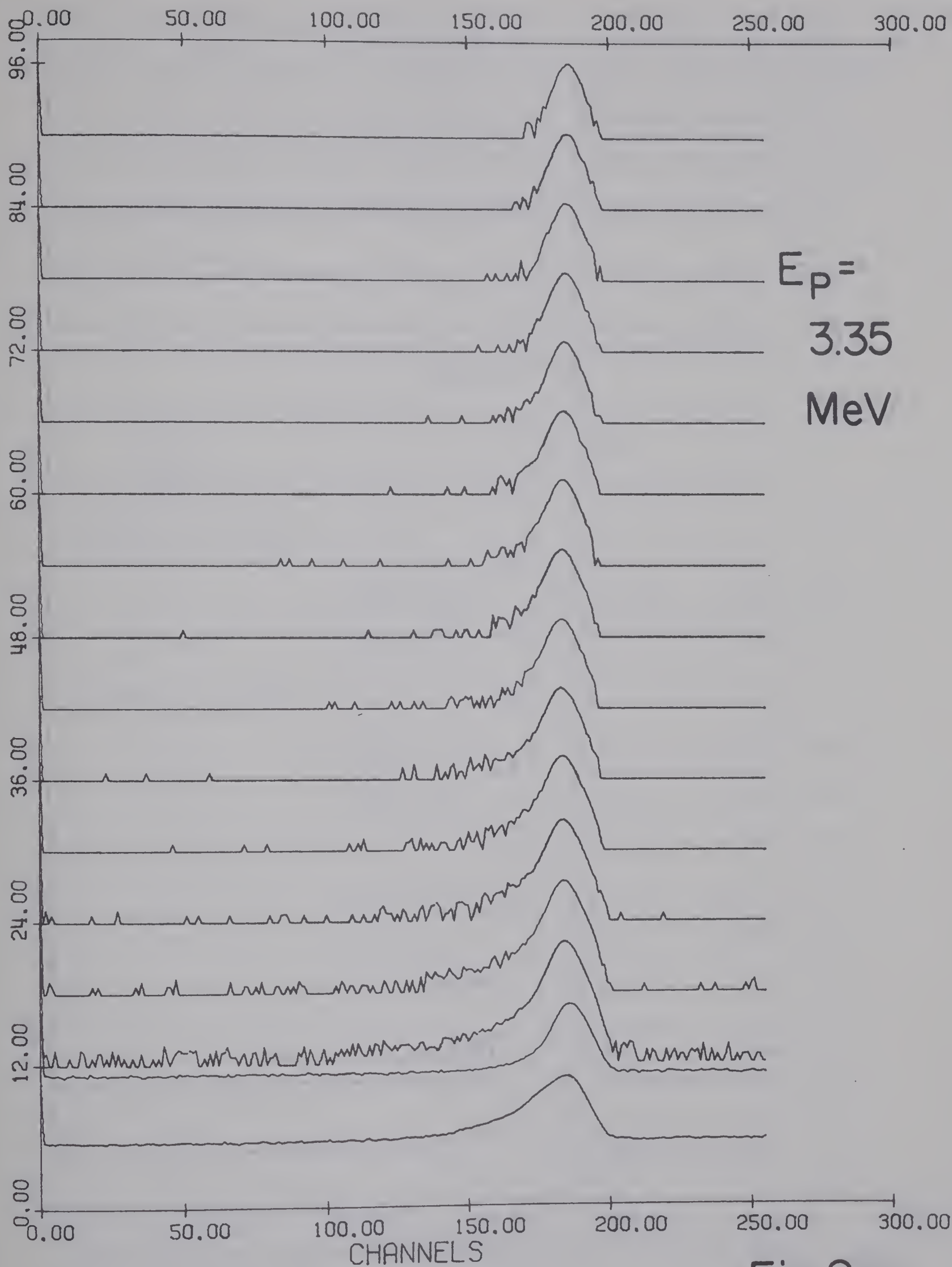


Fig. 9.g

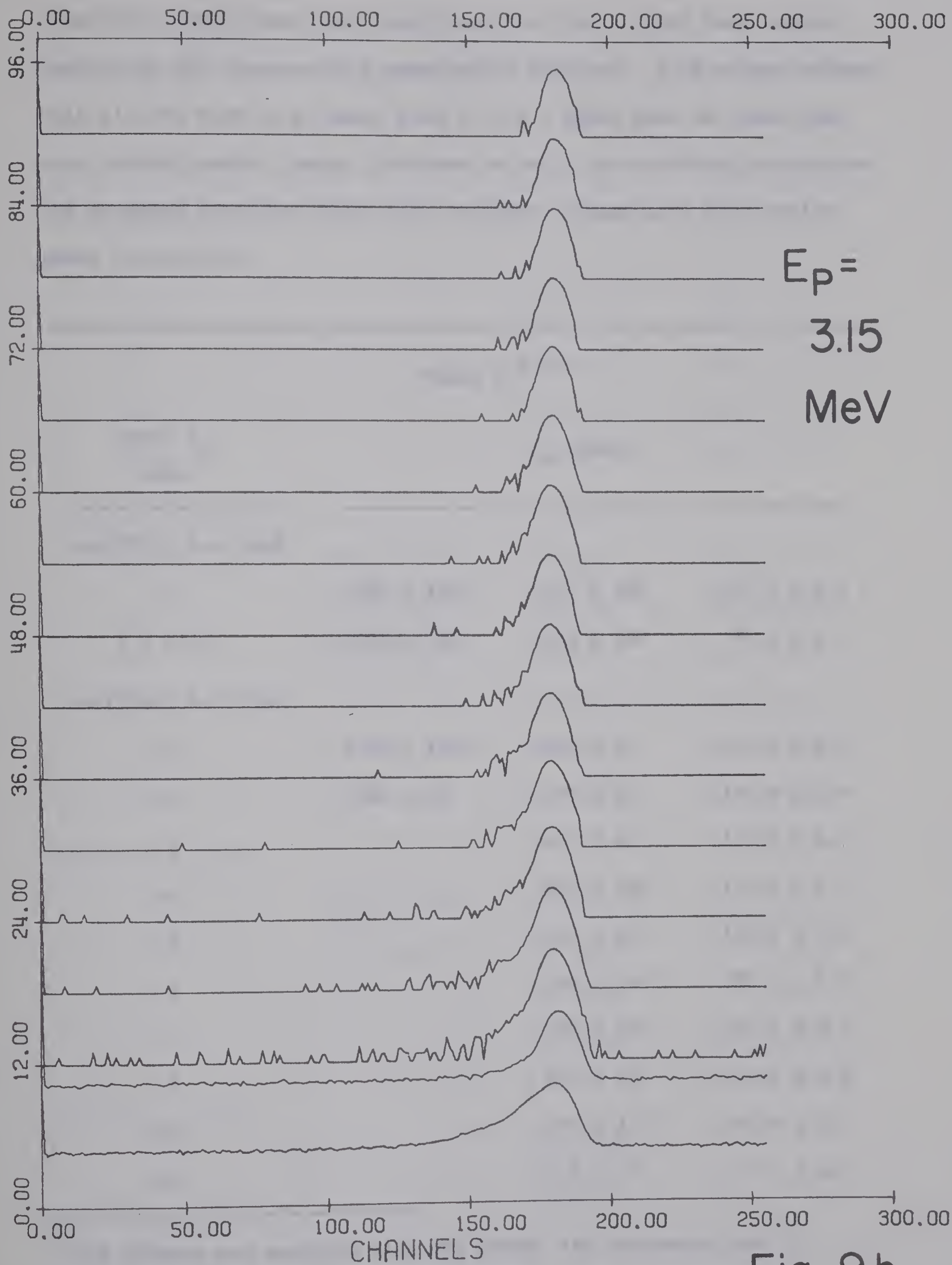


Fig. 9.h

whose left hand slopes were less steep than their right hand slopes indicating the presence of a measureable lifetime. Such slopes existed with all the runs in at least bins 0 to 6. Since most of these bins also had noticeable longer lifetimes as well, the unfolding procedures and programs described above were employed. These gave the results shown in table IV.

TABLE IV* **

Run; E _p Bins	τ _m (psec)		
4/6/70/3; 4.45 MeV			
1	7500 <u>±</u> 1200	470 <u>±</u> 100	104.7 <u>±</u> 0.9
∑ 4 - 15	2800 <u>±</u> 780	420 <u>±</u> 160	97.4 <u>±</u> 1.6
4/6/70/4; 4.45 MeV			
1	6100 <u>±</u> 1300	594 <u>±</u> 27	166.1 <u>±</u> 0.6
2	1265 <u>±</u> 82	406 <u>±</u> 41	154.8 <u>±</u> 0.8
3		690 <u>±</u> 46	170.6 <u>±</u> 1.7
4		920 <u>±</u> 150	174.2 <u>±</u> 1.7
5		707 <u>±</u> 56	158.8 <u>±</u> 2.1
6		750 <u>±</u> 110	182.7 <u>±</u> 6.0
7		760 <u>±</u> 190	147.4 <u>±</u> 3.4
8		621 <u>±</u> 73	152.9 <u>±</u> 3.8
11		600 <u>±</u> 130	140.6 <u>±</u> 5.1
14		231 <u>±</u> 17	117 <u>±</u> 11

*Bin numbers are assigned with the lowest bin designated bin 0.

**Errors include only statistical errors.

4/6/70/5; 4.55 MeV

2	5100 ± 980	482 ± 42	180.9 ± 1.0
3	8800 ± 3700	556 ± 59	198.2 ± 1.3
5		656 ± 52	191.1 ± 1.5
6		776 ± 99	195.8 ± 3.5
7		647 ± 84	181.8 ± 4.1
8		607 ± 85	186.0 ± 4.3
11		475 ± 69	186.6 ± 6.0
13		469 ± 85	161.4 ± 7.7
14		279 ± 22	164.7 ± 2.0

4/6/70/6; 4.25 MeV

2	3800 ± 1000	442 ± 33	153.8 ± 1.2
3	3800 ± 1400	468 ± 47	161.8 ± 1.6
4	4400 ± 1700	467 ± 75	161.7 ± 3.1
5		520 ± 29	158.0 ± 3.5
6		744 ± 98	166.1 ± 4.8
11		930 ± 250	163.6 ± 5.6

4/6/70/7; 3.35 MeV

2	4000 ± 1300	573 ± 75	134.5 ± 1.7
3	2460 ± 980	455 ± 92	110.7 ± 2.8
4		710 ± 110	144.2 ± 2.3
5		890 ± 200	144.0 ± 3.3

4/6/70/8; 3.15 MeV

2	480 ± 42	
3	571 ± 71	

The longest lifetimes, those in the 2500 to 6000 psec range, are due to attempts to fit straight lines to the very longest lifetime tails and as the statistics are very poor in that region, the errors are correspondingly large. Thus, with the longest lifetimes uncertain and the shortest ones being interfered with by the still-present prompt peak, only one lifetime can be inferred from the data. This is the 565 ± 20 psec lifetime found in all the runs and bins up to bin 13 inclusive. This gamma ray is between 2.8 and 3.3 MeV in energy according to the ADCB calibration and the position of its Compton edge ^{approximately} (bin ~~13~~¹¹).
13).

The origin of this gamma ray is now considered. It was established above that the only contributing reactions are $^{45}\text{Sc}(p,p')^{45}\text{Sc}^*$ and $^{45}\text{Sc}(p,n)^{45}\text{Ti}^*$. If the reaction exciting the 565 psec lifetime level were the (p,n) reaction, the gamma ray in question would not be seen at all if the proton bombarding energy were below the threshold for the creation of that particular state (cf. figure 8). Above threshold, the gamma ray would be seen at every bombarding energy due to the continuum of proton energy caused by straggling in the target material; and its appearance as the bombarding energy crossed the threshold would be quite dramatic. As the "565 psec gamma ray" is evident in run 4/6/70/8 ($E_p = 3.15$ MeV) which is just above threshold for the 40 keV level, this would imply that the 40 keV level has a 565 psec mean lifetime if, in fact, the (p,n) reaction were responsible for exciting the lifetime state. However, the gamma ray in question is approximately 3 MeV in energy as mentioned above. This implies that the $^{45}\text{Sc}(p,n)^{45}\text{Ti}^*$ reaction did not feed the level having this measureable mean lifetime and

it is therefore concluded that one of the levels of ^{45}Sc between 2.8 and 3.2 MeV has a lifetime of 565 ± 20 psec. The Ge(Li) detectors used for these runs had very low efficiencies above 2.5 MeV and their spectra could not be used as an aid to further identification. As a result, to determine precisely which level has the measureable lifetime will require a further study perhaps aided by the refinements to be discussed in section V.

From the above discussion, it may also be concluded that the ^{45}Ti has no strongly (p,n) excited levels with mean lifetimes longer than 180 psec (from run/bin 4/6/70/5/2) up to the 1.52 MeV level of excitation.

V. CONCLUSIONS

The data and arguments of section IV give an upper limit of 180 psec on the mean lifetimes of any strongly excited levels of ^{45}Ti up to 1.52 MeV excitation energy excited by the $^{45}\text{Sc}(p,n)^{45}\text{Ti}^*$ reaction. In addition, a new lifetime of 565 ± 20 psec has been assigned to the excitation region of ^{45}Sc between 2.8 and 3.2 MeV and modifications of the above technique to positively identify the responsible level are discussed below.

While no evidence of the previously published²⁷⁾ 1100 psec lifetime level at 376 keV in ^{45}Sc was found, it was carefully looked for. However, the analysis showed that the tail of the 565 psec gamma ray merging into the background fully masked the region where this lifetime tail should have been found in the lower bins. Probably the intensity of the 1100 psec gamma ray is quite weak. Better statistics may show a separation of the two lifetimes if the longer one is a real feature. Along this line, general improvements to the technique and the analysis are to be discussed.

The major problem with the analysis was the separation and identification of the various components in the lifetime tails. This was due to the poor statistics of the data and the lack of a fine enough energy calibration in the upper bins. The obvious solution to the first problem is to extend data collection time and to use thick targets throughout the experiment (care must be taken with targets

that become substantially strong long-lived radiation sources). However, as the collection times were in the order of thirty minutes for a run and the number of counts in the lifetime tails should be increased manifold, the running time required might become prohibitive. While long runs are out of necessity being considered, it might be well worthwhile to give consideration to particle-gamma coincidence. This would positively identify a gamma ray by gating the collected timing peaks with coincident emitted particles of definite energy. With a knowledge of the level scheme from gamma ray spectroscopy, choosing only those gammas from a level defined by a given outgoing particle energy would allow a lifetime study to be made level by level. In addition, without competing gamma rays from other levels and reactions, the numbers of counts in the lifetime tails required in order to obtain a value for a mean lifetime would be less than the numbers collected for this experiment, small as they were.

The second problem may be overcome with an accurate energy calibration in the higher bins using the ^{12}C first excited state at 4.43 MeV by the $^{12}\text{C}(\text{p},\text{p}')^{12}\text{C}^*$ reaction.

Another problem was noted during analysis. While the particle gamma coincidence technique mentioned above would eliminate the need for a prompt peak, when the noncoincidence technique is used as in this work, a method for removal of the prompt peak is found to be necessary to investigate the shorter lifetimes. From observations while running, drifts in the system which affect the resolution appear to occur at a comparatively slow rate so that one suggestion for obtaining a prompt peak is to place the target material on half the surface of a rapidly rotating cylinder, the other half of which is to be covered by a

material producing only prompt gamma radiation. With the lifetimes being considered in this work, the data from the detector could be gated into the data collection region during the exposure of the "prompt" material to the beam and into a second region for the actual data collection while the target material was exposed. This would allow a nearly simultaneous collection of both the data and a prompt peak under the same beam conditions. Other methods, based on deriving prompt peaks simultaneously with the data, are currently being explored at the Nuclear Research Centre.

Perhaps the greatest aid to this work would be an on-line analysis program utilizing either the Honeywell DDP-516 computer or the SDS-920. This type of program would allow an evaluation of the statistics to be taken during such an experiment and on the basis of the evaluation, the experiment would be extended, modified or even terminated (at a tremendous saving in time and effort to the experimenter).

If on-line analysis cannot be provided (a situation which would result in the ambiguities present in this work) it is suggested that a complete analysis program working from data on magnetic tape be developed with provisions for prompt peak subtraction and multiple lifetime extraction. This program should be based on the unfolding algorithm mentioned in section IV and the resultant APL programs in Appendix III.

In conclusion, it is noted that while the method of finding and evaluating nuclear lifetimes by direct electronic measurement does suffer from statistical problems as performed in this work, the above suggestions should eliminate this problem and make the method much easier to carry out (and more practical with respect to the time

required for data analysis). This, combined with the simplicity of the method and its applicability to a wide range of target nuclei should make it a useful experimental tool.

VI. APPENDIX

APPENDIX I^{A.1})

The vector potential $\underline{A}$ which satisfies the wave-equation in free space

$$\Delta \underline{A} - \frac{1}{c^2} \frac{\partial^2 \underline{A}}{\partial t^2} = 0 \quad , \quad \Delta \equiv \nabla^2 \quad [I.1]$$

can be used to derive expressions for an electromagnetic field including the electromagnetic field of γ -rays emitted by excited nuclei. By eliminating the time dependent part of the scalar potential the electromagnetic waves are seen to be transverse from

$$\text{div } \underline{A} = 0 \quad . \quad [I.2]$$

The electric and magnetic fields are given by

$$\underline{E} = - \frac{1}{c} \frac{\partial \underline{A}}{\partial t} \quad [I.3.a]$$

$$\underline{H} = \text{curl } \underline{A} \quad [I.3.b]$$

In quantizing the radiation field the vector potential is expanded as a linear combination of the simultaneous eigenfunctions of equations I.1 and I.2. By confining the radiation to a specified volume and imposing boundary conditions any vector potential $\underline{A}$ which satisfies the boundary conditions can be written as a linear combination of orthogonal waves.

$$\underline{A} = \sum_{\lambda} (q_{\lambda}(t) \underline{A}_{\lambda}(\underline{r}) + q_{\lambda}^*(t) \underline{A}_{\lambda}^*(\underline{r})) \quad [I.4]$$

Here λ represents the various quantum numbers associated with each wave (q_λ and q_λ^* will be defined below). The $\underline{A}_\lambda$ are simultaneous solutions of the vector Helmholtz equation

$$\Delta \underline{A}_\lambda + k_\lambda^2 \underline{A}_\lambda = 0 \quad [I.5]$$

(k_λ being the eigenvalue wave number characterizing $\underline{A}_\lambda$) and of the divergence condition

$$\text{div } \underline{A}_\lambda = 0 \quad [I.6]$$

The $\underline{A}_\lambda$ are normalized for convenience to

$$\int_V \underline{A}_\lambda^*(\underline{r}) \cdot \underline{A}_\lambda(\underline{r}) dV = 4\pi c^2 \delta_{\lambda, \lambda'} \quad [I.7]$$

A single divergenceless vector Helmholtz equation can be derived by combining I.5 and I.6 to get

$$\text{curl curl } \underline{A}_\lambda - k_\lambda^2 \underline{A}_\lambda = 0 \quad [I.8]$$

Solution of this equation will be discussed below.

The field amplitudes q_λ and q_λ^* arbitrarily satisfy the harmonic oscillator equation

$$\frac{d^2 q_\lambda}{dt^2} + \omega^2 q_\lambda = 0 \quad [I.9]$$

where $\omega = ck_\lambda$ is the frequency. The q_λ and q_λ^* represent annihilation and creation operators respectively apart from a multiplicative time constant and a harmonic time factor. These are normalized to obey the commutation rules

$$q_\lambda q_{\lambda'} - q_{\lambda'} q_\lambda = 0 \quad [I.10.a]$$

$$q_{\lambda}^* q_{\lambda}, - q_{\lambda}, q_{\lambda}^* = 0 \quad \text{A3} \quad [I.10.b]$$

$$q_{\lambda} q_{\lambda}^* - q_{\lambda}^* q_{\lambda} = \left(\frac{\hbar}{2\omega_{\lambda}}\right) \delta_{\lambda, \lambda} \quad [I.10.c]$$

The matrix elements of q_{λ} and q_{λ}^* taken between an initial state with n_i quanta characterized by quantum numbers λ and a final state containing n_f quanta also characterized by λ are given by

$$\langle n_{f, \lambda} | q_{\lambda} | n_{i, \lambda} \rangle = n_i \hbar / 2\omega_{\lambda}^{1/2} \delta_{n_f, n_i-1} \exp(-i\omega_{\lambda} t) \quad [I.11.a]$$

$$\langle n_{f, \lambda} | q_{\lambda}^* | n_{i, \lambda} \rangle = (n_i+1) \hbar / 2\omega_{\lambda}^{1/2} \delta_{n_f, n_i+1} \exp(i\omega_{\lambda} t) \quad [I.11.b]$$

The radiation field energy eigenvalues are now found in the following way. The above expressions are used to change the radiation field Hamiltonian

$$H = (8\pi)^{-1} \int_V |\underline{E}|^2 + |\underline{H}|^2 dV \quad [I.12]$$

into

$$H = \sum_{\lambda} 2\omega_{\lambda}^2 q_{\lambda}^* q_{\lambda} \quad [I.13]$$

Thus, from I.10.a and I.10.b we get

$$E_n = \sum_{\lambda} n_{\lambda} \hbar \omega_{\lambda} \quad [I.14]$$

if the order of the field amplitudes q_{λ} and q_{λ}^* are chosen so that there is zero energy when no quanta are present.

In the following work, the vector potential is expanded in a series of standing spherical waves centred at 0. A sphere of radius R_0 is chosen as the volume of integration with the boundary conditions that

the tangential components of $\underline{A}$ vanish at the surface of the sphere. The vector potential for each spherical wave is constant throughout the volume of the sphere and decreases as r^{-1} for large r from the centre. Each of the waves has a definite angular momentum L and corresponds to the outgoing spherical wave emitted by a suitably oscillating 2^L pole at 0. A box quantization with plane waves is not used as the plane waves do not have definite angular momentum and the classical multipole oscillator correspondence is also missing. Before finding a solution to the divergenceless vector Helmholtz equation (equation II.7), solutions to the scalar Helmholtz equation

$$\Delta u + k^2 u = 0 \quad [I.15]$$

will be discussed. It can be shown that

$$u_{LM} = j_L(k_\lambda r) Y_{LM}(\theta, \phi) \quad [I.16]$$

with integer $L \geq 0$ and $|M| \leq L$, satisfies I.13. The $Y_{LM}(\theta, \phi)$ are the normalized spherical harmonics and the $j_L(k_\lambda r)$ are the spherical Bessel functions

$$j_L(k_\lambda r) = \left(\frac{\pi}{2k_\lambda r}\right)^{1/2} J_{L+1/2}(k_\lambda r) \quad [I.17]$$

When $u(R_0) = 0$ is imposed as a boundary condition, $j_L(k_\lambda R_0) = 0$ which requires

$$k_\lambda = (n + (L/2))R_0 \quad [I.18]$$

where n is a positive integer for $R_0 \gg L\pi/2k$.

Since solutions to the vector Helmholtz equation will be expressed in terms of the u_{LM} 's several characteristics of the latter will now be noted.

- 1) The u_{LM} form a complete orthogonal set of non-singular solu-

tions to the scalar Helmholtz equation with the orthogonality condition

$$\int_V u_{LM}^* u_{LM} dV = (R_0/2k_\lambda^2) \delta_{L'L} \delta_{M'M} \delta_{\lambda,\lambda} \quad [I.19]$$

2) Defining

$$\underline{L} = -i \underline{r} \times \text{grad}, \quad [I.20]$$

the u_{LM} satisfy

$$L_z u_{LM} = M u_{LM} \quad [I.21.a]$$

$$\underline{L}^2 u_{LM} = L(L+1) u_{LM} . \quad [I.21.b]$$

This implies that the u_{LM} are eigenfunctions of the orbital angular momentum operators.

$$3) \quad u_{LM}(-\underline{r}) = (-1)^L u_{LM}(\underline{r}) \quad [I.22]$$

The u_{LM} thus have even (+1) or odd (-1) parity depending upon the value of L . The solutions to the divergenceless vector Helmholtz equation can be expressed in terms of the u_{LM} 's as mentioned above, but for each u_{LM} two linearly independent solutions A_{LM}^σ are chosen where σ is either E or M denoting electric or magnetic multipole fields.

$$\underline{A}_{LM}^E = k_\lambda^{-1} C_L \text{curl } \underline{L} u_{LM} \quad [I.23.a]$$

$$\underline{A}_{LM}^M = i C_L \underline{L} u_{LM} . \quad [I.23.b]$$

C_L is an arbitrary constant and

$$\left[\frac{\partial}{\partial \underline{r}} (\underline{r} u_{LM}) \right]_{R_0} = 0 \quad [I.24]$$

The $\underline{A}_{LM}$ have properties similar to the u_{LM} .

1) The $\underline{A}_{LM}$ form a complete orthogonal set of non-singular solutions to the divergenceless vector Helmholtz equation. The orthogonality condition is

$$\int_V (\underline{A}_{L'M'}^{\sigma})^* \cdot \underline{A}_{LM}^{\sigma} dV = \frac{L(L+1)}{2k_{\lambda}^2} R_o C_L^2 \delta_{\sigma'\sigma} \delta_{L'L} \delta_{M'M} \delta_{k_{\lambda}k_{\lambda}} \quad [I.25]$$

Equation I.7, the normalization condition is satisfied if

$$C_L = \left[\frac{8\pi\omega_{\lambda}^2}{L(L+1)R_o} \right]^{1/2} \quad [I.26]$$

$$2) \text{ Defining } \underline{J} = \underline{L} + \underline{S} \quad [I.27]$$

and

$$S_z \underline{A} = i \underline{\varepsilon}_z \times \underline{A} \quad [I.28]$$

where $\underline{\varepsilon}_z$ is a unit vector in the z direction and S_z is the z component of intrinsic spin, the $\underline{A}_{LM}$ satisfy

$$\underline{J} \underline{A}_{LM}^{\sigma} = M \underline{A}_{LM}^{\sigma} \quad [I.29.a]$$

$$\underline{J}^2 \underline{A}_{LM}^{\sigma} = L(L+1) \underline{A}_{LM}^{\sigma} \quad [I.29.b]$$

$$\underline{S}^2 \underline{A}_{LM}^{\sigma} = S(S+1) \underline{A}_{LM}^{\sigma} = 2 \underline{A}_{LM}^{\sigma} \quad [I.29.c]$$

This implies that the electromagnetic field associated with $\underline{A}_{LM}^{\sigma}$ contains a total angular momentum $\underline{L}$ with component M in the z direction.

3) The $\underline{A}_{LM}^{\sigma}$ differ from the u_{LM} in that the former have the division into electric and magnetic multipoles. The electric and magnetic fields, $\underline{E}$ and $\underline{H}$, are related to the time independent vector potentials by

$$\underline{E}_{LM} = ik_{\lambda} \underline{A}_{LM}^{\sigma} \quad \text{A7} \quad [I.30.a]$$

$$\underline{H}_{LM} = \text{curl } \underline{A}_{LM}^{\sigma} \quad [I.30.b]$$

From this it follows that

$$\underline{E}_{LM}^E = \underline{H}_{LM}^M = iC_L \text{curl } \underline{u}_{LM} \quad [I.31.a]$$

$$\underline{E}_{LM}^M = -\underline{H}_{LM}^E = -k_{\lambda} C_L \underline{u}_{LM} \quad [I.31.b]$$

which satisfy the equation

$$\underline{r} \cdot \underline{E}_{LM}^M = \underline{r} \cdot \underline{H}_{LM}^E = 0 \quad [I.32]$$

APPENDIX II^{A.2)}

The development of the quantum mechanical transition probability from perturbation theory starts with the total Hamiltonian

$$H = H_0 + H' \quad [\text{II.1}]$$

being the sum of an unperturbed Hamiltonian H_0 with normalized eigenfunctions u_n and eigenvalues E_n

$$H_0 u_n = E_n u_n, \quad [\text{II.2}]$$

and a small time dependent perturbation H' . Stationary solutions of the Schrodinger equation do not exist with the total Hamiltonian and the time-dependent equation

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \quad [\text{II.3}]$$

must be used. In order to solve this equation, ψ is expressed as an expansion in the eigenfunctions of the unperturbed time-dependent equation using time dependent expansion coefficients.

$$\psi = \sum a_n(t) u_n e^{-iE_n t/\hbar} \quad [\text{II.4}]$$

where $\sum$ is to be taken as a summation over the discrete set of eigenfunctions and integration over the continuous set. Putting II.4 into II.3 gives

$$\sum i\hbar \dot{a}_n u_n e^{-iE_n t/\hbar} + \sum a_n E_n u_n e^{-iE_n t/\hbar} = \sum a_n (H_0 + H') u_n e^{-iE_n t/\hbar} \quad [\text{II.5}]$$

Replacing $H_0 u_n$ with $E_n u_n$, left multiplying by $\bar{u}_k$, and integrating over all space gives

$$i\hbar \dot{a}_k e^{-iE_k t/\hbar} = \sum_n a_n e^{-iE_n t/\hbar} \int_{\tau} \bar{u}_k H' u_n d\tau. \quad [\text{II.6}]$$

For convenience

$$\int_{\tau} \bar{u}_k H' u_n d\tau \equiv H'_{kn} \quad [\text{II.7}]$$

which is called the matrix element of the perturbation. Further defining

$$\omega_{kn} = \frac{E_k - E_n}{\hbar} \quad [\text{II.8}]$$

equation II.6 becomes

$$\dot{a}_k = (i\hbar)^{-1} \sum_n H'_{kn} a_n e^{i\omega_{kn} t} \quad [\text{II.9}]$$

The perturbation approximation consists in replacing H' by $\lambda H'$ in II.1 and II.9 and by expressing the a 's as a power series in λ

$$a_n = a_n^{(0)} + \lambda a_n^{(1)} + \lambda^2 a_n^{(2)} + \dots \quad [\text{II.10}]$$

Equating coefficients of equal powers of λ and setting $\lambda = 1$ the set of equations results

$$\dot{a}_k^{(0)} = 0; \quad \dot{a}_k^{(s+1)} = (i\hbar)^{-1} \sum_n H'_{kn} a_n^{(s)} e^{i\omega_{kn} t}, \quad s = 0, 1, 2, \dots \quad [\text{II.11}]$$

The first equation for $\dot{a}_k$'s shows that the $a_k^{(0)}$ are constant in time. Their values give the initial conditions of the problem before the perturbation is applied. It is now assumed that all but one of the $a_k^{(0)}$ are zero, implying that the system is in a definite unperturbed energy state at the time the perturbation is first applied. Thus

$$a_k^{(0)} = \delta_{km} \quad \text{or} \quad a_k^{(0)} = \delta(k - m) \quad [\text{II.12}]$$

depending upon the discreteness of the state m . Using II.12 the first order equation gives

$$a_k^{(1)}(t) = (i\hbar)^{-1} \int_{-\infty}^t H'_{km}(t') e^{-i\omega_{km}t'} dt' \quad [\text{II.13}]$$

When H'_{km} is of finite duration, the amplitude of a state u_k ($k \neq m$) (after the perturbation has disappeared) is proportional to the time Fourier component of H'_{km} that corresponds to ω_{km} . If H'_{km} is independent of time except for being turned on at time 0 and off at time t , the first order amplitudes at the time t is

$$a_k^{(1)}(t) = -\frac{H'_{km}}{\hbar} \frac{e^{i\omega_{km}t} - 1}{\omega_{km}} \quad [\text{II.14}]$$

Thus, if the system is originally in state m , the probability of finding the system in state k at time t is

$$|a_k^{(1)}(t)|^2 = \frac{4|H'_{km}|^2 \sin^2(\omega_{km}t/2)}{\hbar^2 \omega_{km}^2} \quad [\text{II.15}]$$

From the plot of $\sin^2 \left(\frac{\omega_{km}t}{2\omega_{km}} \right)$ v. s. ω_{km} (cf. A.2) it can be seen that the probability of finding the system in one of the groups of states k ($E_k \sim E_m$) is proportional to t if H'_{km} is approximately independent of k .

The transition probability per unit time, W , is found by assuming the system to be in a cubical box of dimension x with periodic boundary conditions at the walls. The eigenfunctions u_n now form a discrete set and are normalized to unity in the volume x^3 . A group of final states k with $E_k \sim E_m$ are considered and it is assumed that H'_{km} varies slowly with

k. The density of final states

$$\frac{dN}{dE} \equiv \rho(k) \quad [\text{II.16}]$$

is assumed to a slowly varying function of k also. Using this, the transition probability per unit time to some state k of the final group is

$$W = t^{-1} \sum_k |a_k^{(1)}(t)|^2 = t^{-1} \int |a_k^{(1)}(t)|^2 \rho(k) dE_k \quad [\text{II.17}]$$

where the integration is introduced by taking x large enough. The H'_{km} and $\rho(k)$ are taken outside the integral as they are slowly varying and most of the contribution comes from a small energy range about E_m , giving

$$W = \frac{1}{t} \frac{4 |H'_{km}|^2}{\hbar} \rho(k) \int_{-\infty}^{\infty} \frac{\sin^2(\omega_{km} t/2)}{2 \omega_{km}} d\omega_{km} \quad [\text{II.18}]$$

Since $\omega_{km} \approx 0$, the integral can be rewritten as

$$\frac{t}{2} \int_{-\infty}^{\infty} x^{-2} \sin^2 x dx = \frac{\pi t}{2} \quad [\text{II.19}]$$

so that

$$W = \frac{2\pi}{\hbar} \rho(k) |H'_{km}|^2 \quad [\text{II.20.a}]$$

or

$$\equiv \frac{2\pi}{\hbar} |\langle f | H' | i \rangle|^2 \frac{dN}{dE} \quad [\text{II.20.b}]$$

using the notation of equation 6, section II.

APPENDIX III^{A.3)}

```

V LP;X;Y;Z
[1] →0×10≠p[]←((p,A)≠p,C) / 'NO. OF CHANNELS ≠ NO. OF COUNTS'

[2] X←LSTSQ
[3] Y←Q(3,pC)pA,(⊕C),X[2]+X[1]×B×A-1/A
[4] Z←[],0p[]←CAR,'DAY/MONTH/YEAR/RUN/BIN NUMBER : '
[5] 40 60 PLOT Y
[6] '
      DATA POINT = ○    THEORETICAL = *'
[7] 'C:  ';C

```

```

V RES←LSTSQ;AA;N;W;R;X
[1] AA←B×A-1/A
[2] W←((2ppC)p1,(pC)p0)×((2ppC)pC)
[3] R←X+.×W+.×QX←(2,pC)pAA,(pC)p1
[4] R←SSOLVE1 R
[5] 'A AND B OF Y=AX + B (COUNTS = A(TIME) + B) ARE: ';
    RFS←R+.×X+.×W+.×⊕C
[6] 'MEAN LIFE AND ERROR ARE:  ';1÷RES[1];'  ';(R[1;1]*
    0.5)÷RES[1]*2
[7] 'INVERSE NORMAL MATRIX IS:  ';R

```

SSOLVE1 IS A MATRIX INVERSION


```

V ERROR;S;AAA;N;R
[1] S←STANDEV
[2] AAA←B×1ρS
[3] H← 2 2 ρ1
[4] H[1;1]←+/(1÷(S*2))
[5] H[1;2]←H[2;1]←+/(AAA÷(S*2))
[6] H[2;2]←+/(AAA*2)÷(S*2))
[7] R←SSOLVE1 H
[8] 'N: ' ;N
[9] 'R: ' ;R
[10] C←P-Q
[11] A←1ρC
[12] RES←LSTSQ
[13] 'CORRECTED ERROR IN THE SLOPE IS: ' ;(R[2;2]*
    0.5)÷RES[1]*2
V

```

```

V S←STANDEV;AAA;N;R
[1] 'INPUT LINEAR PRKS: '
[2] P←[]
[3] 'INPUT ANTILN OF CONTINUED LONGER LIFETIME: '
[4] Q←[]
[5] →0×10≠ρ[]←((ρ,P)≠ρ,Q)/'ρP≠ρQ'
[6] 'STANDARD DEVIATIONS OF LN OF DIFF ARE: '
[7] []←S←((P+Q)*0.5)÷(P-Q)
V

```



```

      V ERROR3;S;AAA;N;R
[1] S←STANDE3
[2] AAA←R×ipS
[3] N← 2 2 p1
[4] N[1;1]←+/(1÷(S*2))
[5] N[1;2]←N[2;1]←+/(AAA:(S*2))
[6] N[2;2]←+/(AAA*2)÷(S*2))
[7] R←SSOLVE1 N
[8] 'N: ' ;N
[9] 'R: ' ;R
[10] C←(P-Q)-PP
[11] A←ipC
[12] RES←LSTSC
[13] 'CORRECTED ERROR IN THE SLOPE IS: ' ;(R[2;2]*
      0.5)÷RES[1]*2

```

V

```

      V S←STANDE3
[1] 'INPUT PRKS'
[2] P←[
[3] 'INPUT EXTENSTION OF LONGEST'
[4] Q←[
[5] 'INPUT EXFFNSTION OF SECOND LONGEST'
[6] PP←[
[7] P←S+((P+Q+PP)*0.5)÷((P-Q)-PP)

```

V

VII. GLOSSARY

Analogue to Digital Converter: ADC, an electronic device which assigns digital information to an input analogue signal proportional to the input signal size.

Compton edge: the drop off in counts with increasing energy in an energy spectrum due to the maximum allowable value of the energy transfer in Compton scattering

$$T_{\max} = \frac{h\nu_o}{1 + \frac{1}{2 \cdot \frac{h\nu_o}{mc^2}}}$$

A point two-thirds of the way down the Compton edge is considered $T_{\max}$.

continuum: the energy region where the discrete energy states of a nucleus are so close together that their widths overlap.

de-excite: to lose energy as in a nucleus changing from a higher excited nuclear state to a lower one.

Doppler shift: the change of frequency of an oscillating wave due to motion of the source, observer or both.

electric multipole : A single point charge is a monopole. A dipole is two opposite charges separated by a small distance s . A quadrupole is obtained by displacing a dipole a distance s_1 and placing a dipole (separation distance s_2) of opposite

sign in the original dipole position such that $s_1 = s_2$. This is continued for higher poles.

electric multipole radiation: radiation of the form given off by an oscillating electric multipole.

excited nucleus: a nucleus whose energy is above its lowest possible value. The energies which can be assumed by the nucleus are discrete up to the continuum (discrete states).

gamma ray: highly energetic electromagnetic radiation (wavelength $\lesssim 10^{-10}$ m).

Ge(Li) detector: lithium drifted germanium solid state depletion region detector possessing excellent energy resolution (3 ~ 4 keV).

Hamiltonian: quantum mechanical expression for the total energy of a system.

magnetic multipole: a concept arising from the scalar magnetic potential

$$|V_m| = \left| \frac{\mu_0}{4\pi} I \Omega \right|$$

where I is the current and Ω the solid angle subtended by the circuit at the field point at which a magnetic induction, $\underline{B}$, is to be measured,

$$|\underline{B}| = |\nabla V_m|.$$

Then

$$V_m = \frac{\mu_0}{4\pi} \int_{\tau} \frac{\underline{M} \cdot \underline{r}}{r^2} d\tau$$

where τ is a small volume and $\underline{M} d\tau$ is the magnetic dipole.

magnetic multipole radiation: radiation of the form given off by an oscillating magnetic multipole.

mean nuclear lifetime: τ_m , the time taken for a number of excited nuclei to decay so that $1/e$ of the original number of excited nuclei still survive in the excited state.

nanosecond: nsec, 10^{-9} second.

nuclear halflife: $\tau_{1/2}$, the time taken for a number of excited nuclei to decay so that $1/2$ of the original number of excited nuclei still survive in the excited state. $\tau_{1/2} = 0.693147 \tau_m$

nuclear model: a mathematic-physical concept of the nucleus which attempts to present as accurate a picture as possible of the true nuclear form (which is as yet uncertain).

nuclear state: either the lowest (ground) or one of the excited energy levels of a nucleus (cf. excited nucleus) characterized by various quantum numbers (spin, parity, isotopic spin).

parent, daughter nucleus: When a nucleus gives off a particle and becomes a different nucleus, the original is termed the parent and the resulting nucleus is termed the daughter.

parity: the parity describes the effect on a system by transposing each coordinate through the space origin. If the system does not change sign it is said to have even parity; if it does change sign it is said to have odd parity.

photomultiplier tube: an electronic device which gives a relatively large electrical output for a very small light input.

picosecond: psec, 10^{-12} second.

scintillator: any material which emits light when struck by an atomic or subatomic particle, or by electromagnetic radiation.

target: the collection of nuclei to be bombarded in a nuclear physics experiment. Usually in solid or gaseous form.

Time to Amplitude Converter: TAC, an electronic device which outputs a signal proportional to the time between the input start and input stop pulses.

timing jitter: uncertainty in timing due to various effects.

transition probability: the probability for a nucleus to change from one excited nuclear state to another.

walk: change in the timing signal due to change of the triggering crossover intercept with pulse height variation.

wavefunction: a mathematical description of a given state of a quantum mechanical system.

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